Well-balanced physics-based finite volume schemes for Saint-Venant-Exner models of sediment transport

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Abstract

The Saint-Venant-Exner (SVE) model is widely used for the description of sediment transport including bedload, erosion, and deposition processes. A modified version of the SVE model, which includes sediment concentration incorporates exchange of sediment between the fluid and an erodible bed and a non-hydrostatic pressure for the fluid along with non-equilibrium entrainment and deposition velocities, is introduced. Gravitational effects on erosion are described by an effective shear stress formulation. This modified SVE model is derived from a general approach with density variations. It preserves the mass of both the sediment and the fluid, and satisfies a dissipative energy balance. On the other hand, well-balanced finite volume schemes adapted for SVE models are derived since standard well-balanced schemes for the Saint-Venant system with fixed bottom are in general no more well-balanced when applied to the SVE model. The latter property is due to the uncontrolled numerical diffusion associated with the bed evolution equation. Two novel techniques to achieve the well-balanced property for the modified SVE model are proposed. The first is a new polynomial-viscosity-matrix-based (PVM) scheme, denoted "PVM-2I", that modifies the numerical approximation of the bed evolution equation according to its related characteristic speed. The second is a physically motivated correction of the numerical diffusion term for the Rusanov and Harten-Lax-van Leer (HLL) schemes. The proposed schemes are positivity-preserving for the water height. Numerical solutions are compared with exact solutions with gravitational effects, with a novel exact solution in non-equilibrium conditions, and with experimental data. It is illustrated how the use of standard non-well-balanced schemes leads to a large artificial (unphysical) erosion and completely degraded solutions. This undesirable behaviour is avoided by the proposed well-balanced schemes.

This presentation is based on joint work with Enrique D. Fernández-Nieto (Universidad de Sevilla, Spain) and Jorge Moya (Universidad de Concepción).

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Acknowledgements

This research has been partially supported by ANID (Chile) through Fondecyt project 1250676, Centro de Modelamiento Matemático CMM (BASAL project FB210005), and CRHIAM, project ANID/ FONDAP/ 1523A0001.

Improved geomodelling in infrastructure projects

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Abstract

In engineering geology, the integration of geological, geophysical, and geotechnical models is essential for accurately representing subsurface conditions in relation to infrastructure development. Such infrastructure may include hydroelectric plants, major roads, or viaducts, where understanding the subsurface is critical for design, safety, and cost-efficiency. This work presents selected examples of data acquisition and modelling processes applied in real-world cases, with the aim of improving model evaluation practices. The examples illustrate how different types of data—ranging from borehole logs and seismic surveys to geotechnical tests—can be synthesized to construct comprehensive subsurface models. The findings suggest that while it is often possible to approximate subsurface conditions with reasonable accuracy, the reliability and consistency of the models are strongly influenced by the quantity and quality of available information. In cases where data coverage is sparse or heterogeneous, discrepancies between models become more pronounced, potentially affecting engineering decisions. Conversely, projects with robust and diverse datasets tend to yield models with higher degrees of agreement and predictive value. The study highlights the importance of early-stage data acquisition planning and interdisciplinary collaboration to enhance model reliability. Ultimately, the work underscores the need for continuous refinement of modelling methodologies and evaluation criteria to support resilient and sustainable infrastructure development.

This work has been performed with support from the Division of Engineering Geology, LTH, Lund University and the Swedish Road Administration.

High order Finite Volume WENO schemes with Invariant-Region-Preserving properties for one-dimension multispecies kinematic flow models

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Abstract

Multispecies kinematic flow models are defined by systems of strongly coupled, nonlinear first-order conservation laws, where the solution is a vector of partial volume fractions or densities. These models arise in various applications including multiclass vehicular traffic and sedimentation of polydisperse suspensions. The solution vector should take values in a set of physically relevant values (i.e., the components are nonnegative and sum up at most to a given maximum value). It is demonstrated that this set, the so-called invariant region, is preserved by numerical solutions produced by a new family of high-order finite volume numerical schemes adapted to this class of models. To achieve this property, and motivated by [X. Zhang, C.-W. Shu, On maximum-principle-satisfying high order schemes for scalar conservation laws, J. Comput. Phys. 229 (2010) 3091–3120], a pair of linear scaling limiters is applied to a high-order weighted essentially non-oscillatory (WENO) polynomial reconstruction to obtain invariant-region-preserving (IRP) high-order polynomial reconstructions. These reconstructions are combined with a local Lax-Friedrichs (LLF) or Harten-Lax-van Leer (HLL) numerical flux to obtain a high-order numerical scheme for the system of conservation laws. It is proved that this scheme satisfies an IRP property under a suitable Courant-Friedrichs-Lewy (CFL) condition. The theoretical analysis is corroborated with numerical simulations for models of multiclass traffic flow and polydisperse sedimentation.

This presentation is based on joint work with Juan David Barajas-Calonge (Universidad del Bío-Bío-Chile), Raimund Bürger (Universidad de Concepción-Chile) and Pep Mulet (Universidad de Valencia-Spain).

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Acknowledgements

This research has been partially supported by ANID (Chile) through Centro de Modelamiento Matemático CMM (BASAL project FB210005).

A second-order invariant-region-preserving scheme for a transport-flow model of polydisperse sedimentation

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Abstract

A polydisperse suspension is a mixture of a number N of species of small solid particles, which may differ in size or density, dispersed in viscous fluid. The sedimentation of such a mixture gives rise to the segregation of species and flow of the mixture due to density fluctuations. In two space dimensions, and for equal-density particles, this process can be described by a hyperbolic system of N nonlinear conservation laws for the particle volume fractions coupled with a version of the Stokes system for the volume-averaged flow field of the mixture. A second-order numerical scheme for this transport-flow model is formulated by combining a finite-difference approximation of the Stokes system with a finite volume (FV) scheme for the transport equations, both defined on a Cartesian grid on a rectangular domain. The FV scheme is based on a central weighted essentially non-oscillatory (CWENO) reconstruction applied to the first-order local Lax-Friedrichs (LLF) numerical flux. By the application of scaling limiters to the CWENO reconstruction polynomials and utilizing that the Stokes solver generates a discretely divergence-free (DDF) velocity field, one can prove that the FV scheme has the invariant region preserving (IRP) property, i.e., the volume fractions are nonnegative and sum up at most to a set maximum value. Numerical examples illustrate the model and the scheme.

This presentation is based on joint work with Raimund Bürger (Universidad de Concepción, Chile), Pep Mulet (Universitat de València, Spain) and Luis Miguel Villada (Universidad del Bío-Bío, Chile).

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Acknowledgements

This research has been partially supported by ANID (Chile) through Becas Doctorado Nacional 2022, folio 21221387.

Modeling Coastal Evolution from an Engineering Perspective

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Abstract

An important part of coastal engineering is mathematical modeling of sediment transport and coastal morphological evolution over a wide range of temporal and spatial scales. The area of interest to model typically encompasses the region where wind-generated waves interact with the bed and the water motion induced by the waves mobilize and transport sediment. In general, this area extends from a water depth of about twice the incident wave height to the runup limit on the subaerial portion of the beach. Traditionally, more detailed models involve simulating waves, currents, sediment transport, and morphological change, where these elements interact in a complex manner and a significant challenge pertains to formulate the coupling between them that maintains the main physics, but still makes simulations possible in a robust and effective manner.

There are many important problems arising from sediment movement in coastal areas, including beach erosion during storms, effects of coastal structures on shoreline evolution, coastal retreat due to sea level rise, harbor siltation, and infilling of navigational channels. Sediment transport occurs at many different scales, inducing gradients that cause the development a wide range of morphological features. At the scale of individual waves over seconds, ripples form, whereas for sea level rise occurring over centuries, some specific properties of the beach profiles is preserved. This range of characteristic scales in sediment transport and morphological evolution have led to the development of a variety of mathematical models where each aims at describing the governing physical mechanisms at a particular scale, treating processes at adjacent scales as boundary conditions (larger) or noise (smaller).

The procedure of mathematical modeling of coastal evolution at an engineering level typically involves three steps: (1) formulation, (2) evaluation, and (3) application. In the first step the governing processes are identified, equations to describe them developed, and a numerical technique selected. Then, model verification, calibration, and validation are performed, including sensitivity analysis and uncertainty estimates. Finally, the developed model can be used for analysis, prediction, and design with regard to the specific problem under consideration.

Here, several different modeling approaches will be discussed considering a variety of engineering problems of interest in coastal evolution, bearing in mind their characteristic scale. The models, as well as the procedures involved to develop and apply them, will be

reviewed starting from simplistic behavior-oriented models to complex physics-based formulations that include all the elements required to simulate waves, currents, sediment transport, and morphological change.

Numerical approximation for a fluid flow problem arising from reverse osmosis modeling in water desalination

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Abstract

We consider a mathematical model for addressing coupled fluid flow problems arising from reverse osmosis modeling in water desalination processes. It consists of the coupled Navier-Stokes/transport equations, with nonlinear conditions across a semipermeable membrane. To solve these sets of partial differential equations, we present two different numerical methods. First, we employ a mixed finite element method able to capture several variables of interest such as the salt concentration level, pressure drop and fluid velocity. Secondly, a conservative hybridizable discontinuous Galerkin method is also employed. Through diverse numerical simulations and a variety of configurations, we illustrate the capability of both methods to accurately capture the behavior of saline water when passing through membrane-based reverse osmosis desalination channels.

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Acknowledgements

This research has been partially supported by ANID (Chile) through Fondecyt project 1240183, Anillo project ACT210087 (Computational Mathematics for Desalination Processes), and Centro de Modelamiento Matemático CMM (BASAL project FB210005).

A multi-phase continuum model for slow sand filtration

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Abstract

A slow sand filter (SSF) consists of a bed of saturated packed sand submerged in water, through which microorganisms are able attach and, together with other organic and inorganic particles, form a biofilm matrix which is essential for the water purification capabilities of the filter. The resulting biofilm consists of a complex ecological system with multiple inter-species interactions subject to various physical effects such as detachment and dispersive transport in the surrounding flowing suspension. However, most well-known mathematical models for SSFs are comprised of linear models with a limited amount of modelled components. For that reason, an accurate and encompassing multi-phase continuum mathematical-ecological framework was introduced for one-dimensional modelling an SSF under varying operating conditions. A novelty of this model is that it distinguishes different physical volumes in the sand filter: the biofilm pores, the biofilm matrix itself and the surrounding flowing suspension. Further efforts have been made to extend this model to the two-dimensional case, studying a Cahn–Hilliard/Stokes system for the biofilm growth in the supernatant water, with aims to extend the numerical method to a Cahn–Hilliard/Stokes/Darcy system accounting for the sand bed.

This presentation is based on joint work with Stefan Diehl (Lund University) and Julio Careaga (University of Groningen).

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Modelling the friction effect in the shallow water moment equations for granular flows

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Abstract

Shallow-water moment equations (SWME) emerge as a generalization of the classical shallow water equations to vertically variable velocity profiles. The SWME are derived from the mass conservation and momentum balance equations under the assumption that the fluid velocity is described by a polynomial expansion in the vertical coordinate. The moments, which are the coefficients of the polynomial expansion, together with the water height and mean velocity are modelled by a coupled system of nonlinear transport equations. In this work, we study the friction term arising in the momentum and moment equations, which results as a remaining term from the treatment of the viscous-stress tensor. We propose a modelling procedure to incorporate general friction terms to the SWME particularly addressing the case of granular flows. Moreover, we develop a finite volume numerical scheme for correctly approximating the stiffness of the friction source terms. Numerical simulations are presented for different models of friction, including the case of wet-dry fronts.

This presentation is based on joint work with Qian Huang (University of Stuttgart) and Julian Koellermeier (Ghent University).

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This research has been partially supported by the Dutch Research Council (NWO) through the ENW Vidi project HiWAVE with file number VI.Vidi.233.066.

Optimization of sequencing batch reactors for wastewater treatment

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Abstract

Sequencing batch reactors (SBRs) are widely used in wastewater treatment. An SBR is a tank containing a certain amount of activated sludge (biomass) and it is operated in several stages and cyclically. During one stage, it is filled with incoming wastewater, whose nutrients are consumed by the biomass under aerobic and anoxic conditions. When aeration and mixing are stopped, sedimentation and compression of the sludge take place in addition to reactions, and purified water is extracted at the surface. During another stage, a small amount of sludge is discharged at the bottom to compensate for biomass growth. A one-dimensional model of an SBR can be formulated as a moving-boundary problem for a degenerating system of convection-diffusion-reaction equations whose unknowns are the concentrations of solid and liquid components. Reactions are modelled by an established activated sludge model (ASM1). By transforming the model to a fixed computational domain, an efficient well-posed semi-implicit numerical scheme has been developed with the favourable properties that concentrations are non-negative and the total solids concentration less than a maximal one.

The presentation will contain ongoing efforts to optimize the process. An efficient numerical scheme forms the cornerstone of minimizing the running costs of a plant model comprising several parallel SBRs, subject to constraints on maximum allowed effluent concentrations. The running costs are mainly the oxygen supply. For given feed input concentrations and volumetric flow to the plant, there are 21 optimization variables consisting of all the time points of starting and ending filling, aeration, extraction, etc., the discharge volumetric flow and the set point of oxygen concentration for a controller during aeration. A key constraint is to have a periodic solution.

Collaborators: Raimund Bürger (Universidad de Concepción), Julio Careaga (University of Groningen), Estefanía Guevara (Universidad Nacional de Colombia sede Manizales), Romel Pineda (Universidad Central del Ecuador).

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Advancing Membrane Technologies for Sustainable Water Treatment: From Concept to Pilot Scale

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Abstract

Addressing global water scarcity requires innovative and scalable solutions for water treatment and reuse. Membrane technology has emerged as a cornerstone in this field, enabling efficient purification across diverse water sources. This work presents three novel concepts developed and tested from laboratory to pilot and full-scale applications.

The first concept focuses on rain and stormwater harvesting using ceramic micro- and ultrafiltration membranes. Initial trials in Lund, Sweden, demonstrated that silicon carbide membranes effectively remove microplastics, micropollutants, and heavy metals from stormwater, producing water suitable for agricultural use. Building on these results, a full-scale installation at an apartment building in Malmö now supplies treated stormwater for toilets and washing machines, reducing drinking water consumption by approximately 40%. This approach highlights the potential for decentralized water reuse in urban environments.

The second concept, direct membrane filtration (DMF), offers an abiotic alternative for municipal wastewater treatment. By combining coagulation, flocculation, microsieving, and membrane filtration, DMF achieves high rejection rates of carbon and phosphorus, while enhancing biogas production. Pilot studies in Lund using PVDF microfiltration membranes led to the installation of a large DMF unit in Fredrikstad, Norway, capable of treating over 90 m^3 of wastewater per day. The process demonstrates the feasibility of energy-neutral or even energy-positive wastewater treatment, contributing to more sustainable municipal water management.

The third concept integrates forward osmosis (FO), membrane distillation (MD), and nanofiltration (NF) for seawater desalination, powered by concentrated solar energy. Developed within the EU Horizon 2020 DESOLINATION project, this system utilizes a thermo-responsive polymer draw solution and waste heat from a solar power plant to drive desalination. Ongoing pilot trials at Lund University are paving the way for large-scale deployment at King Saud University in Saudi Arabia, aiming to reduce operational costs and improve the sustainability of desalination in arid regions.

Together, these concepts demonstrate the versatility and impact of membrane technologies in addressing water scarcity. By progressing from laboratory research to

real-world pilot and full-scale implementations, they offer practical pathways for improving water reuse, resource efficiency, and environmental sustainability.

Acknowledgements

The research is partly funded by the REWAISE "Resilient Water Innovation for Smart Economy" (Project No. 869496) and DESOLINATION (Project No. 101022686) under the European Horizon 2020 programme.