

Example: Let X and Y be independent stochastic variables with $E[X] = 3$, $E[Y] = 4$, $V[X] = 0.5$ and $V[Y] = 0.9$. Determine the expected value and variance of

a) $2X + 10$

b) $5X - 2Y$

Solution:

a)

$$\begin{aligned} E[2X + 10] &= E[2X] + 10 \\ &= 2E[X] + 10 \\ &= 2 \cdot 3 + 10 = 16. \end{aligned}$$

$$\begin{aligned} V[2X + 10] &= C[2X + 10, 2X + 10] \\ &= C[2X, 2X] + C[2X, 10] + C[10, 2X] + C[10, 10] \\ &= V[2X] = 2^2 V[X] \\ &= 4 \cdot 0.5 = 2. \end{aligned}$$

b)

$$\begin{aligned} E[5X - 2Y] &= E[5X] + E[-2Y] \\ &= 5E[X] - 2E[Y] \\ &= 5 \cdot 3 - 2 \cdot 4 = 15 - 8 = 7. \end{aligned}$$

$$\begin{aligned} V[5X - 2Y] &= C[5X - 2Y, 5X - 2Y] \\ &= C[5X, 5X] + C[5X, -2Y] + C[-2Y, 5X] + C[-2Y, -2Y] \\ &= V[5X] + 2C[5X, -2Y] + V[-2Y] \\ &= 5^2 V[X] + (-2)^2 V[Y] \\ &= 25 \cdot 0.5 + 4 \cdot 0.9 = 12.5 + 3.6 = 16.1. \end{aligned}$$