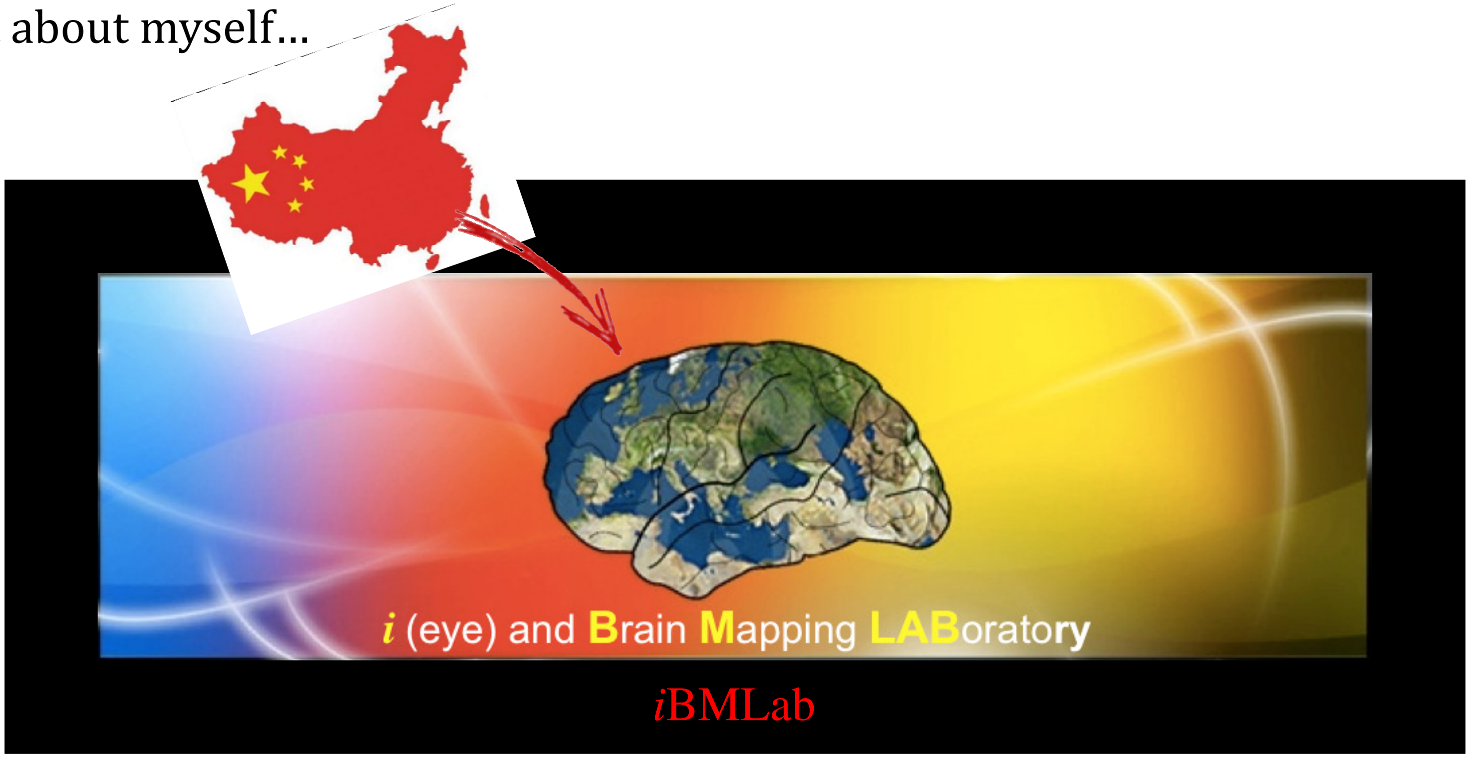
Statistical Inferences of Eye movement data using Bayesian smoothing

Junpeng Lao, PhD

Bayes@Lund, 2017-04-20

I am supported by the Swiss National Science Foundation

Little bit about myself…



University of Glasgow
VIA VERITAS VITA

UNI FR

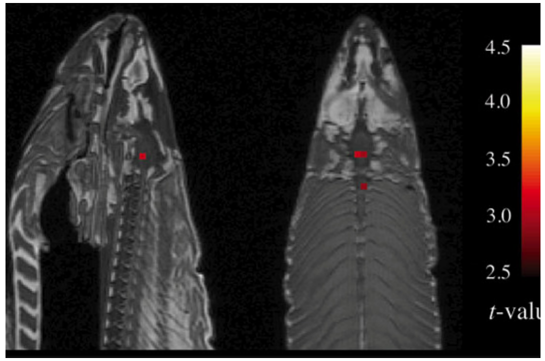
UNIVERSITÉ DE FRIBOURG
UNIVERSITÄT FREIBURG

Little bit about myself…

Eye-tracking

EEG

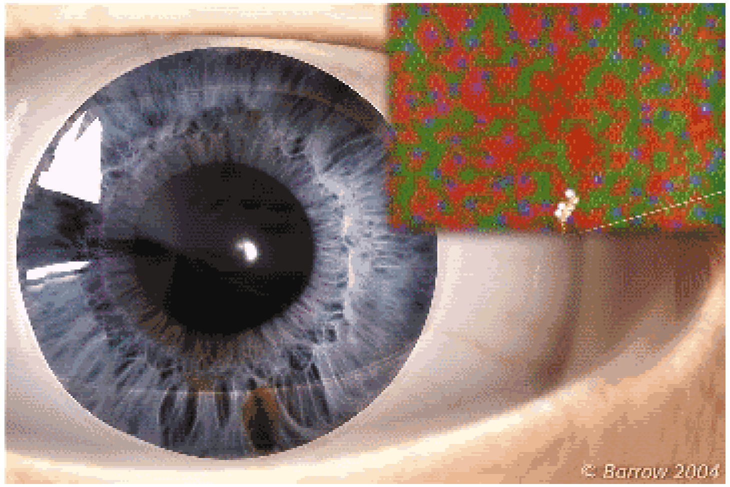
fMRI

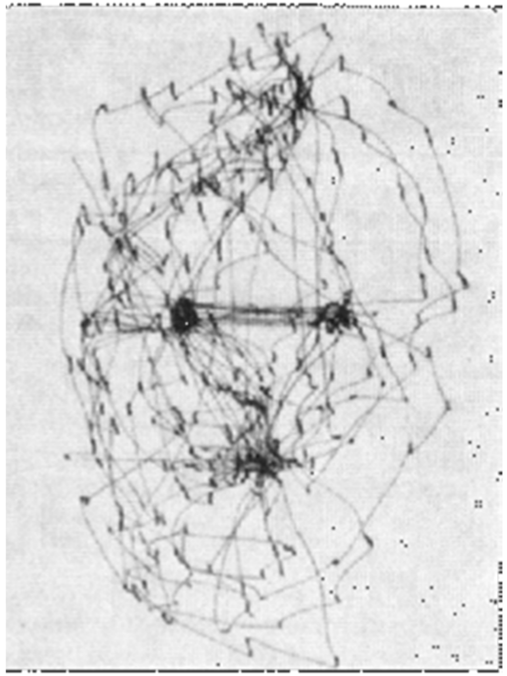


PyMC3

https://github.com/junpenglao/
http://junpenglao.xyz

Eye movement



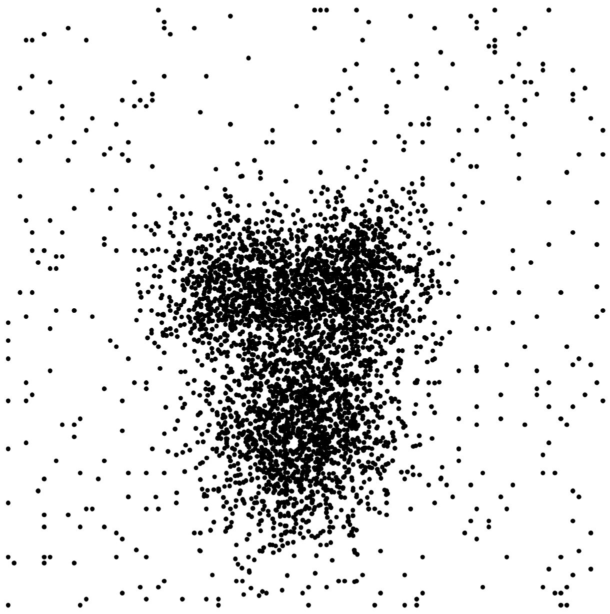


Yarbus, A. L. (1967)

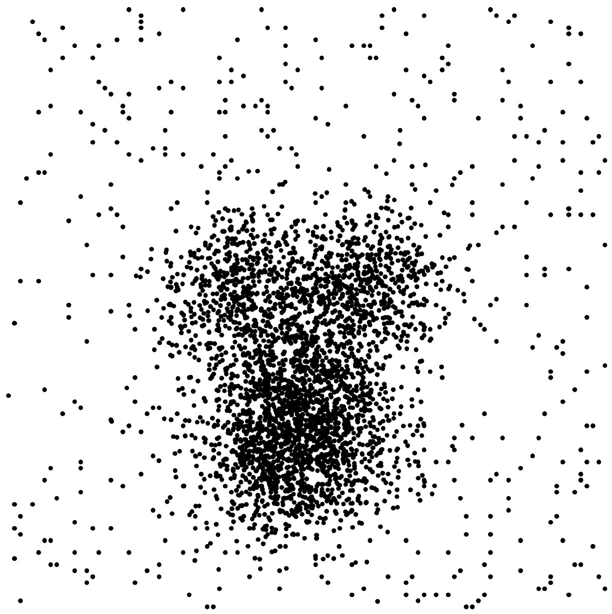
The Question:

Fixations

Group A

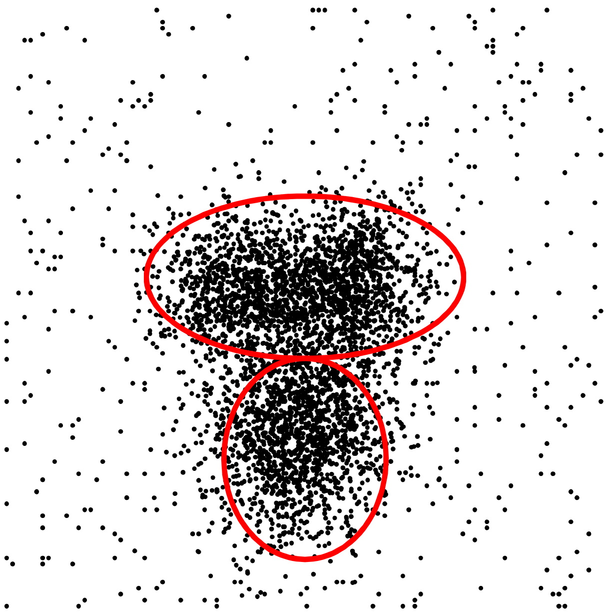


Group B

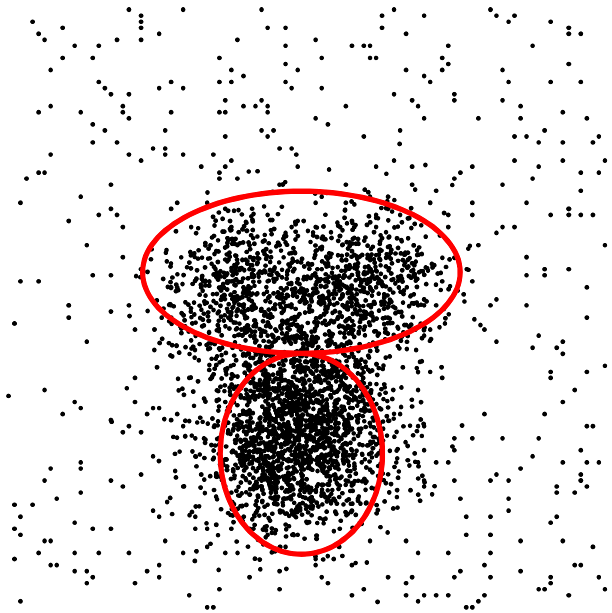


NHST using ROI:

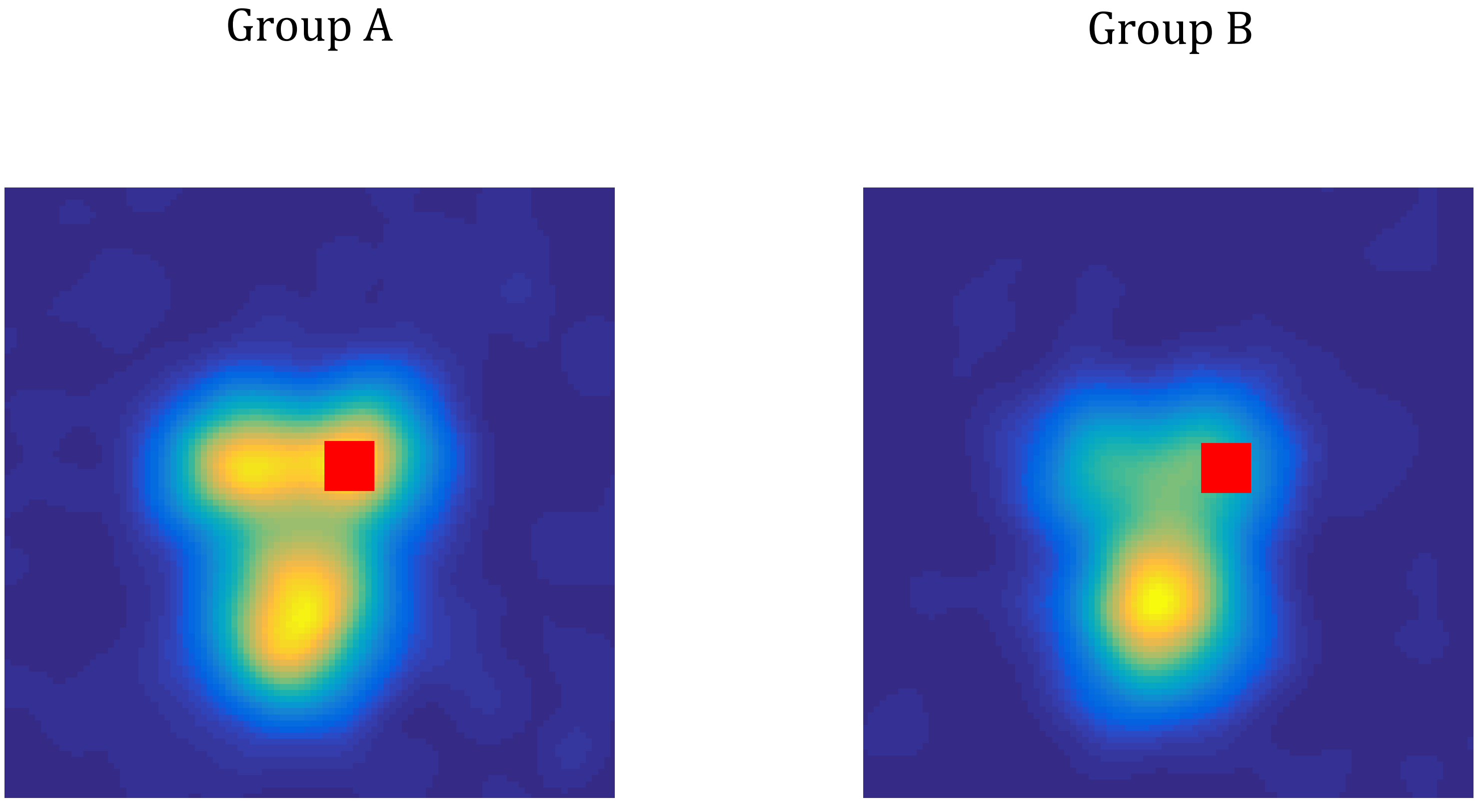
Group A



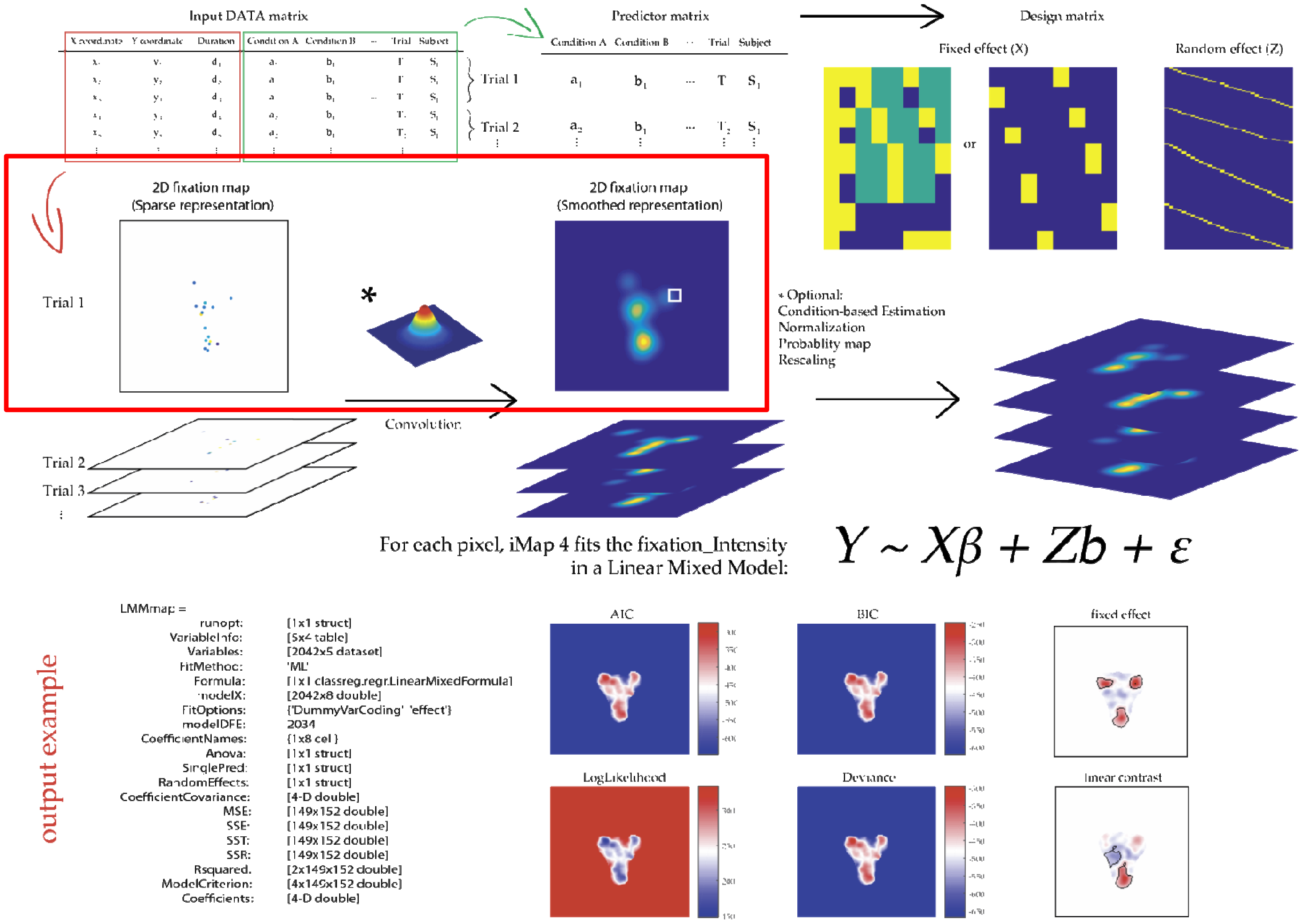
Group B



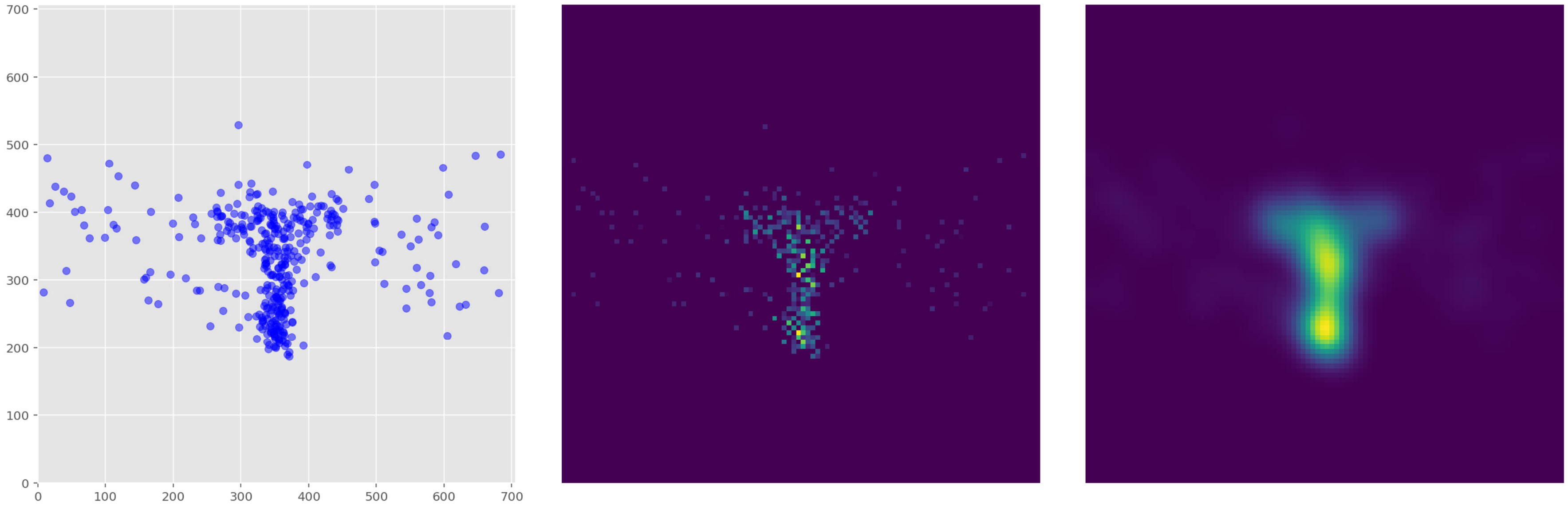
Data-driven analysis



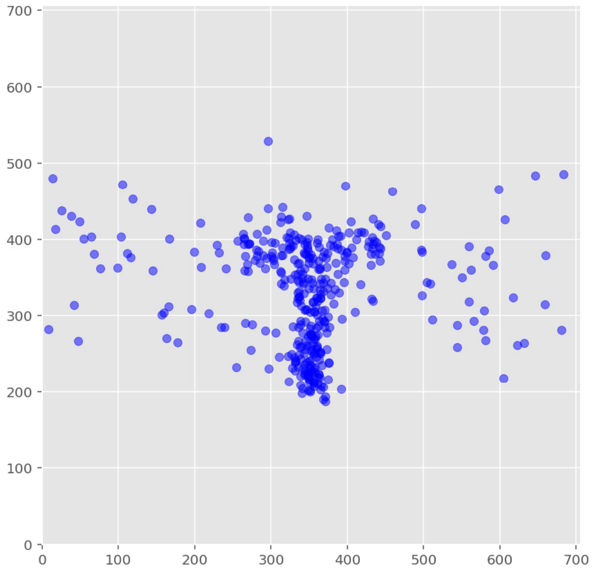
Spatial mapping of fixation data

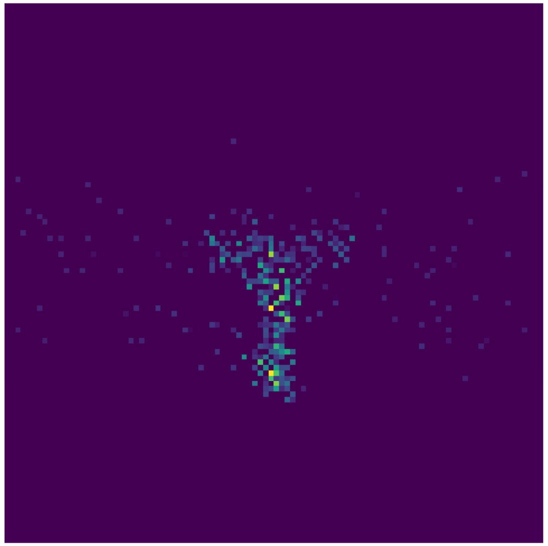


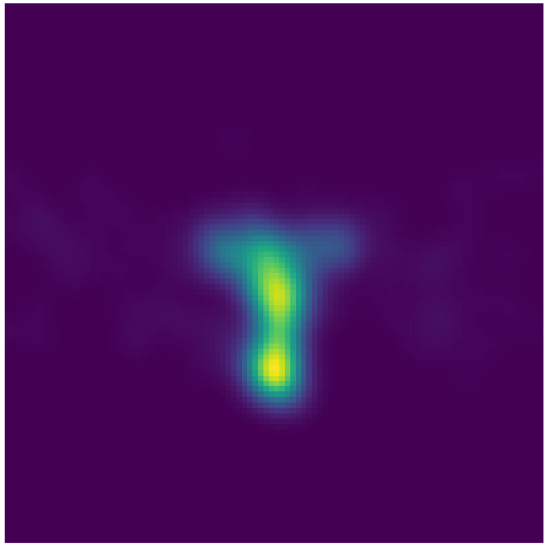
Smoothing

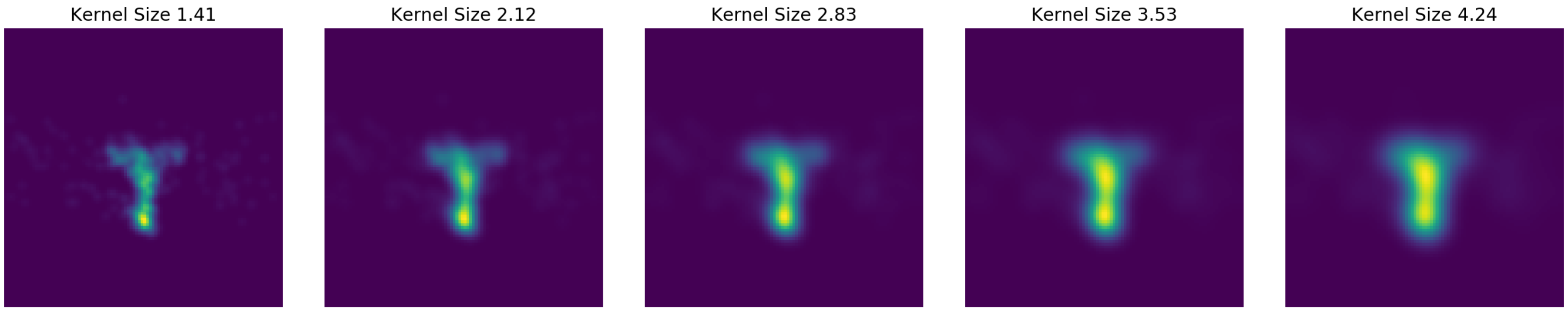


Does size matter?

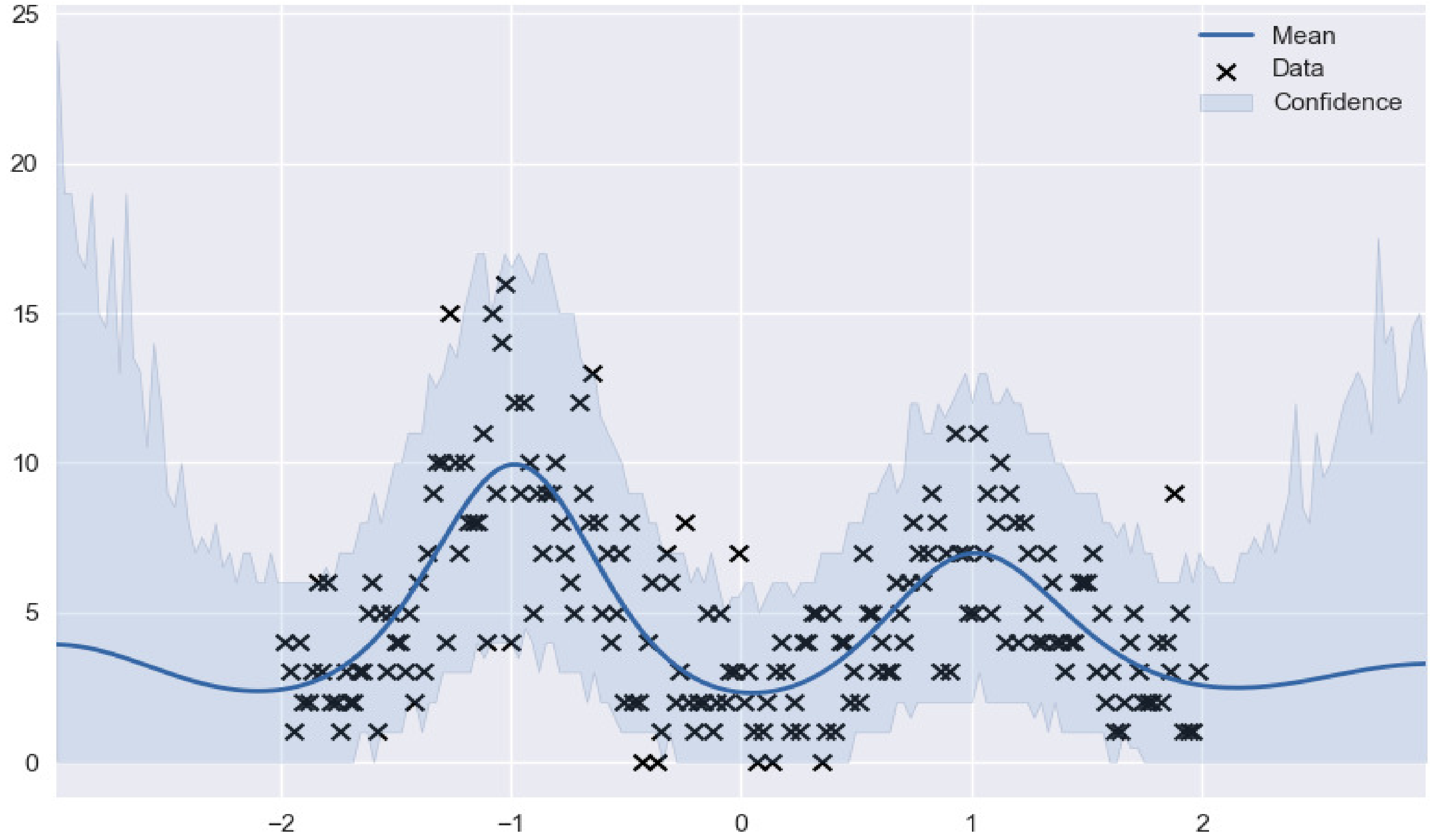




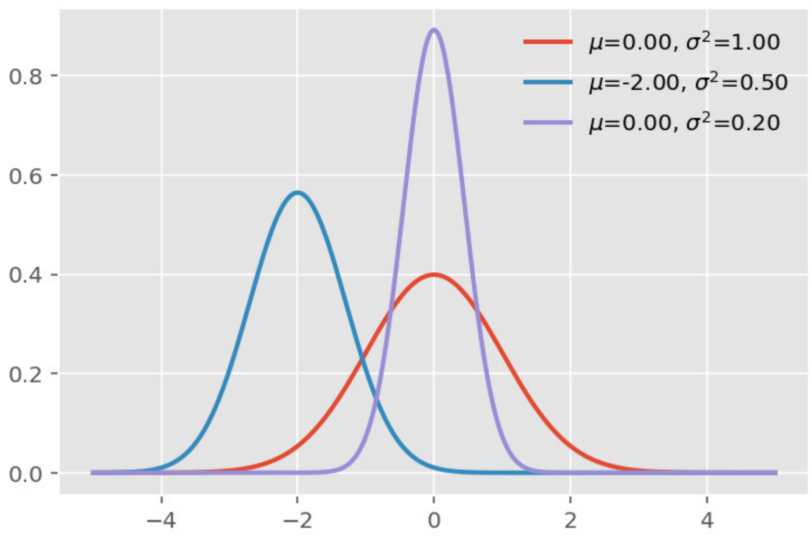


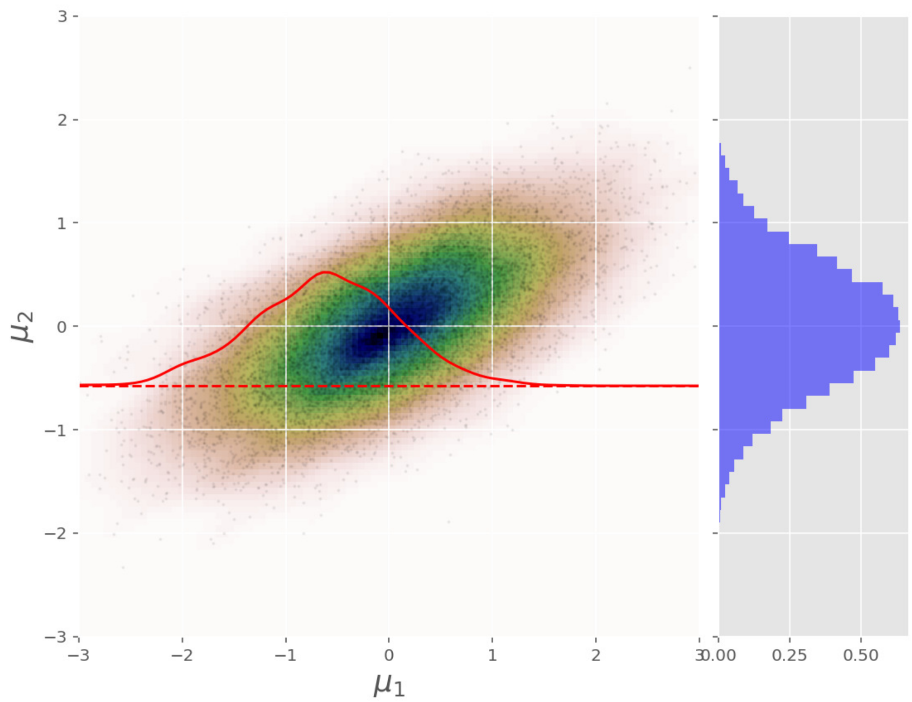


Rethinking



Gaussian Distribution





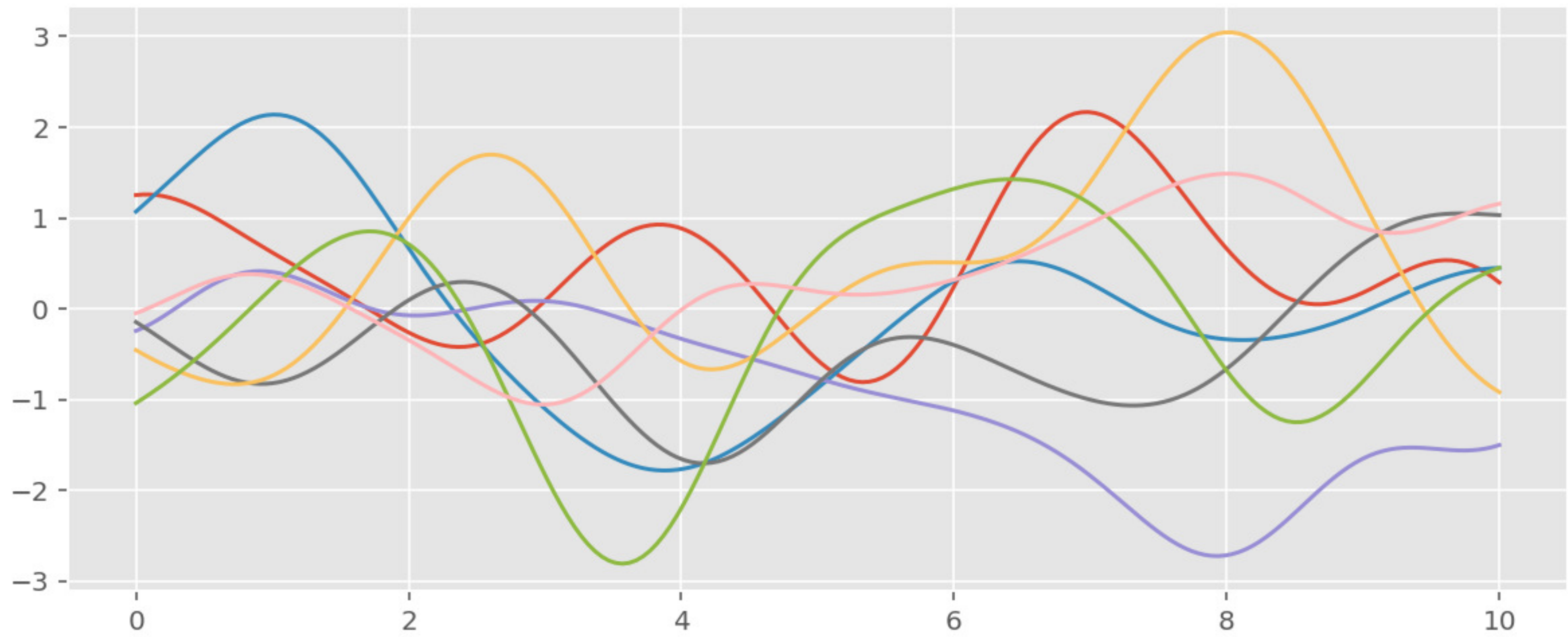
$$p(x \mid \mu, \Sigma) = (2\pi)^{-k/2} |\Sigma|^{-1/2} \exp\left\{ -\frac{1}{2}(x-\mu)'\Sigma^{-1}(x-\mu) \right\}$$

Gaussian Process

Gaussian distribution with infinite dimension

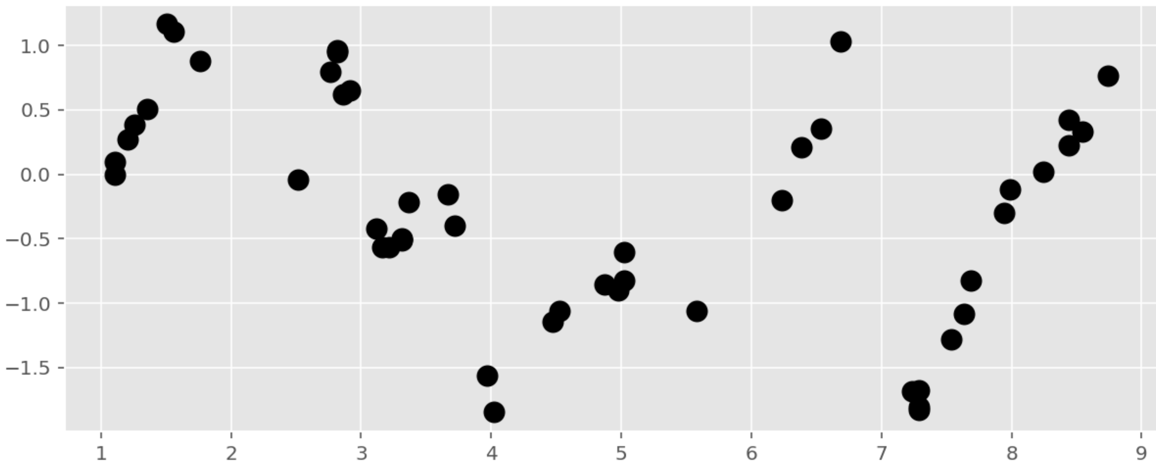
$$p(x) \sim \mathcal{GP}(m(x), k(x, x'))$$

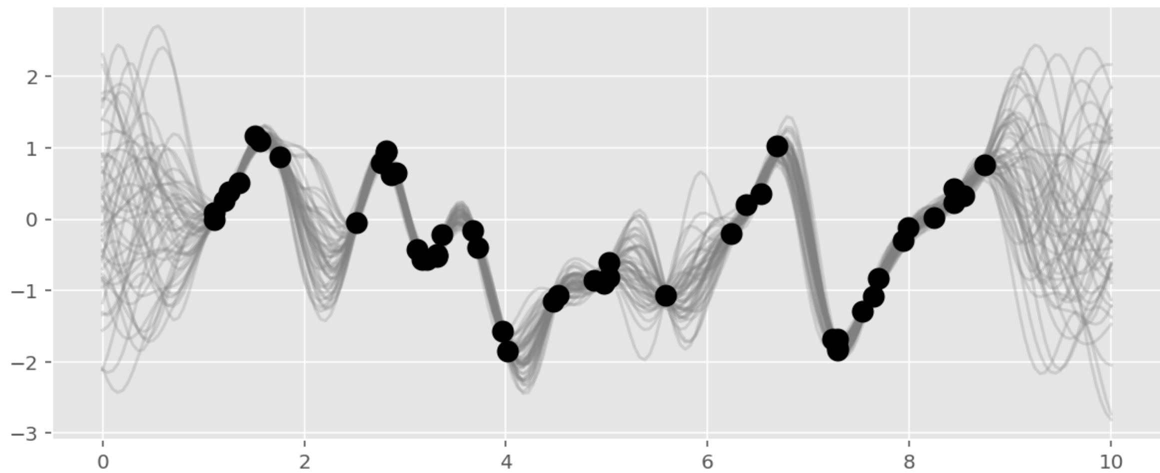
$$p(f(x)) \sim \mathcal{GP}\left(m(x) = 0, k(x, x') = \exp\left(-\frac{1}{2}(x - x')^2\right)\right)$$



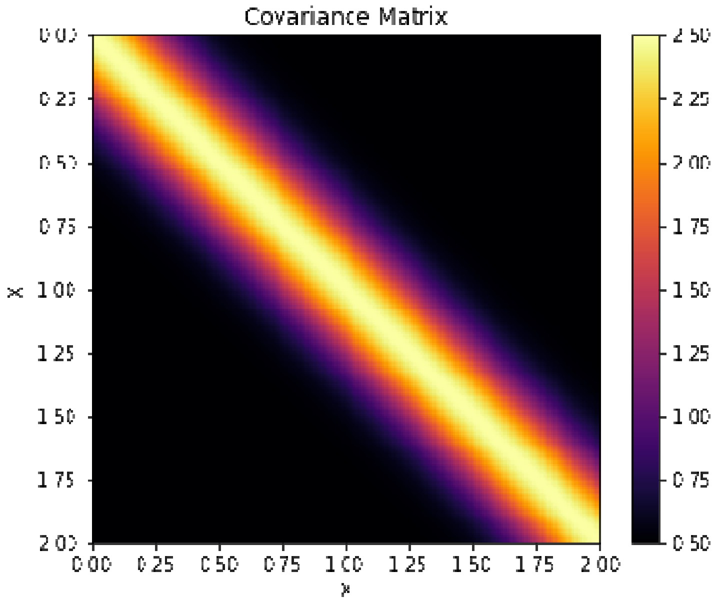
Posterior prediction with GP

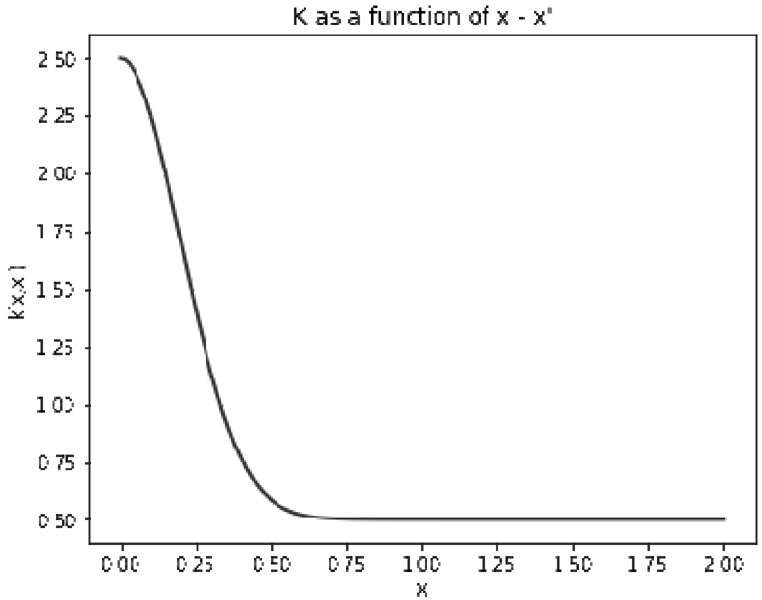
$$f_*|X_*, X, f \sim \mathcal{N}(K(X_*, X)K(X, X)^{-1}f, K(X_*, X_*) - K(X_*, X)K(X, X)^{-1}K(X, X_*))$$

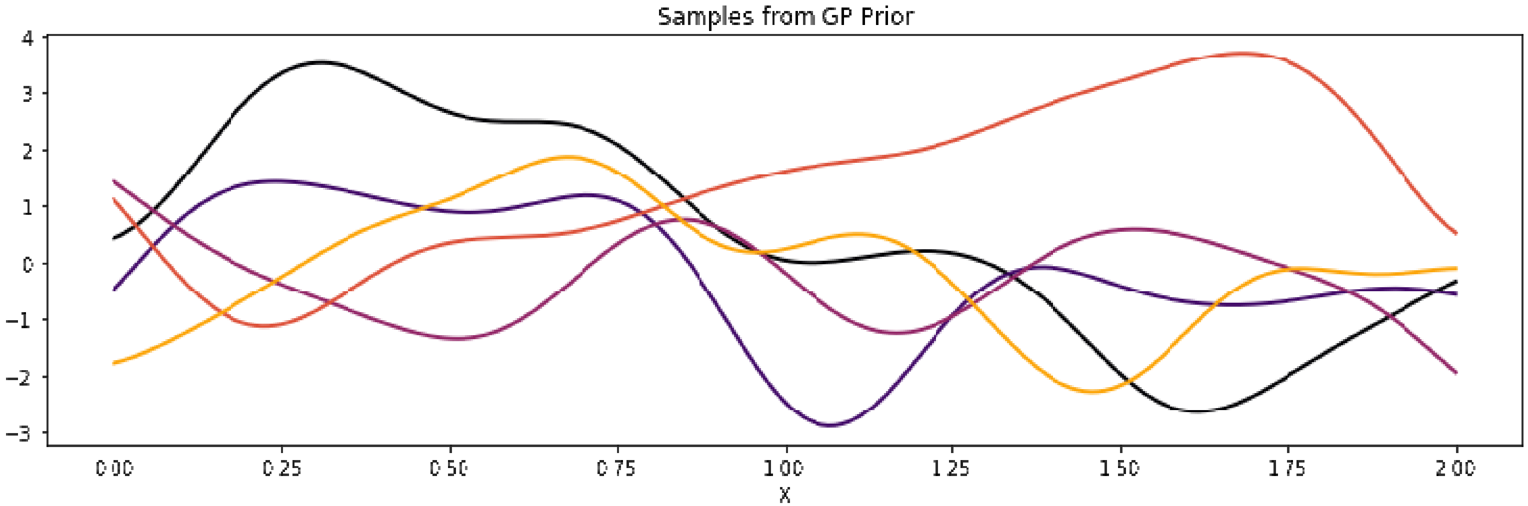




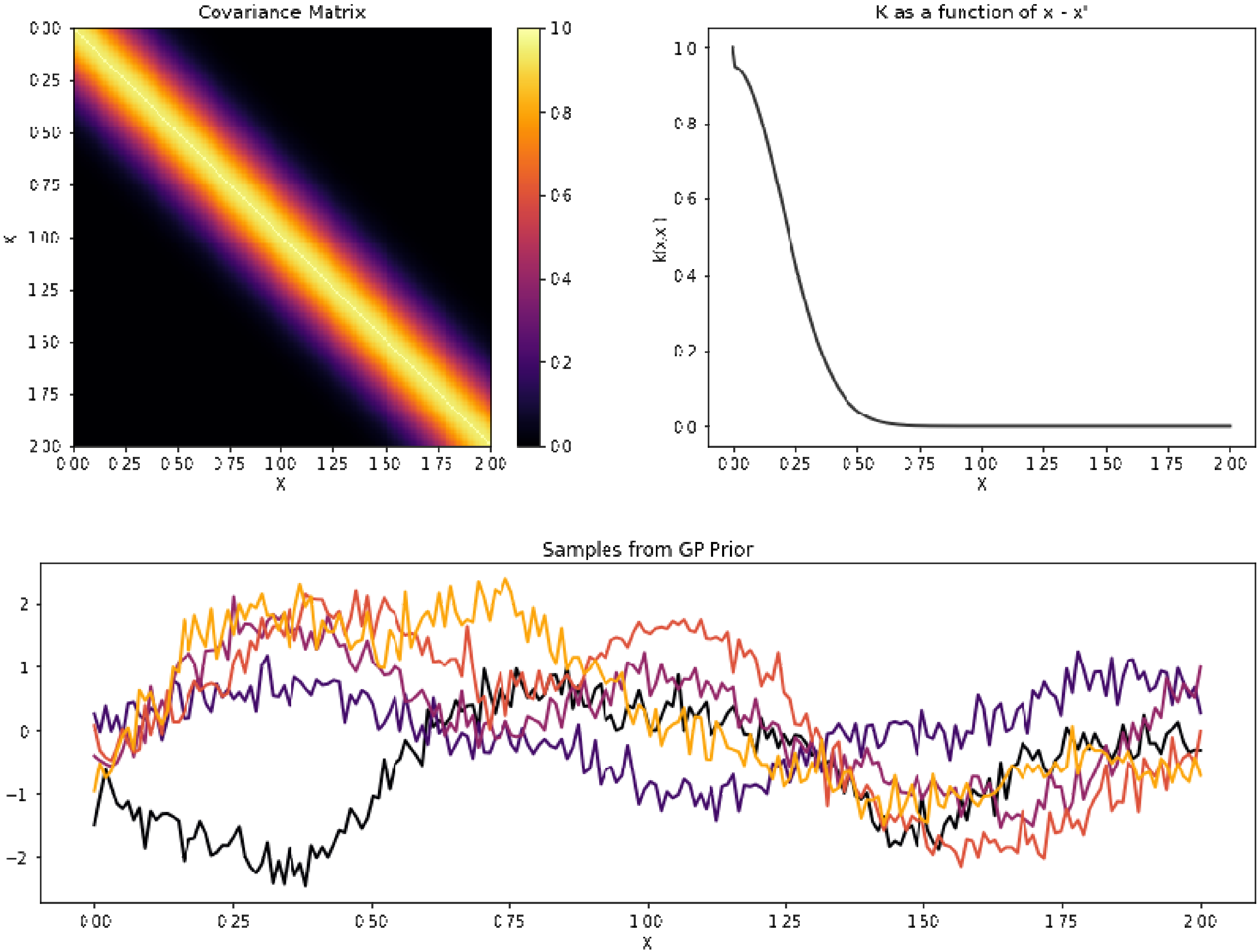
Kernels / Covariance functions





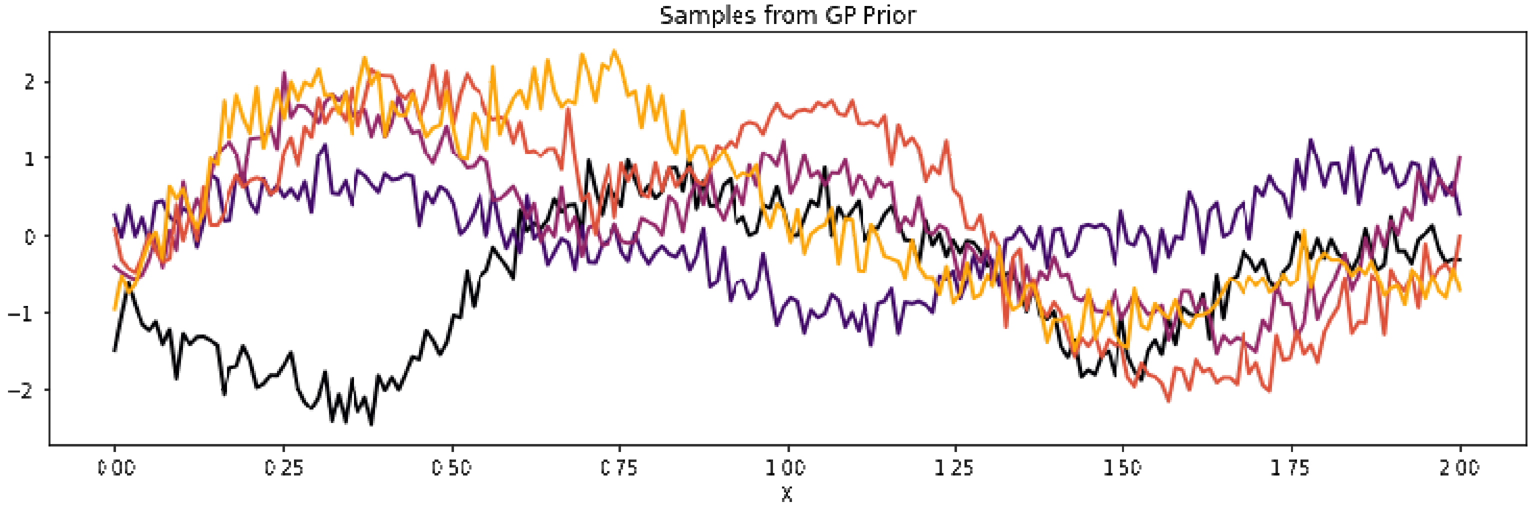


Kernels / Covariance functions



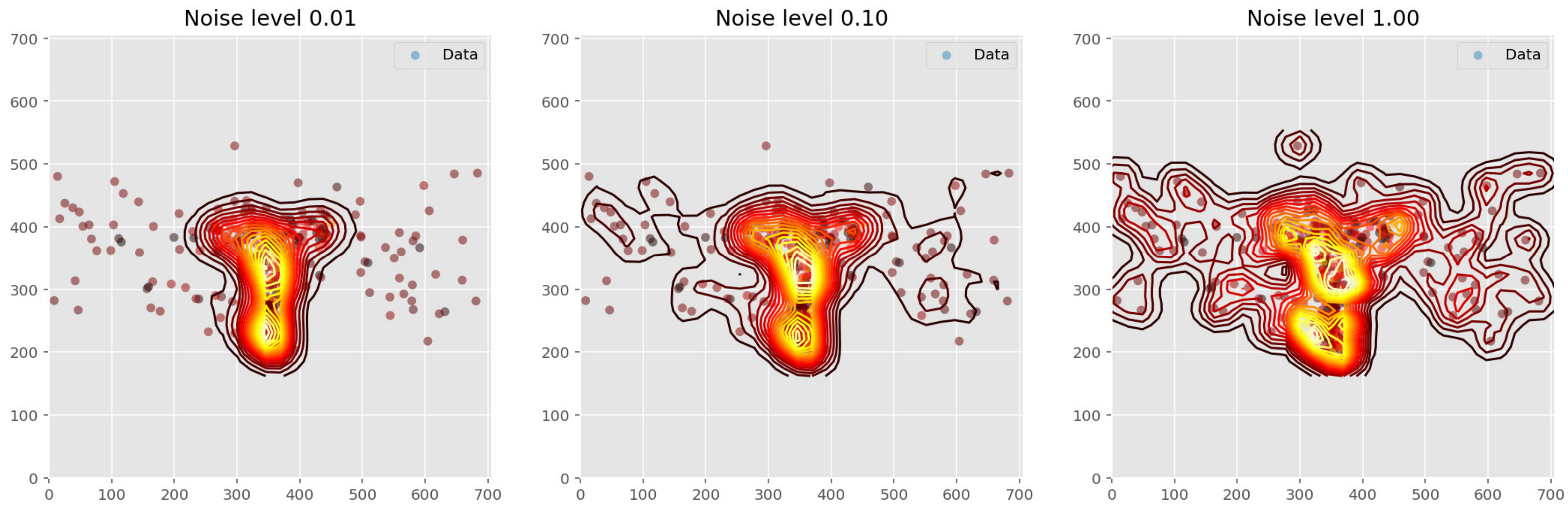
Kernels / Covariance functions

```
cov_latent = tau * pm.gp.cov.ExpQuad(Dim, length_scale)
cov_noise  = sigma2 * tt.eye(n)
Cov_func   = cov_latent(X) + cov_noise)()
```

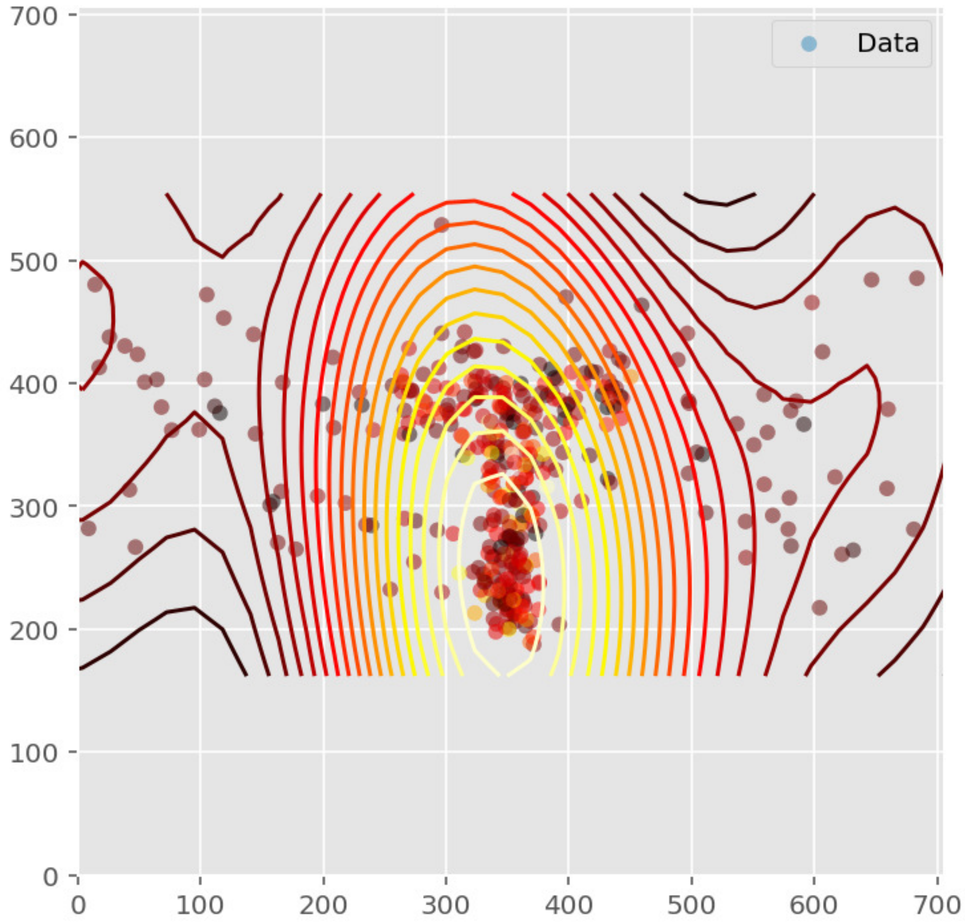


GP regression on fixation data

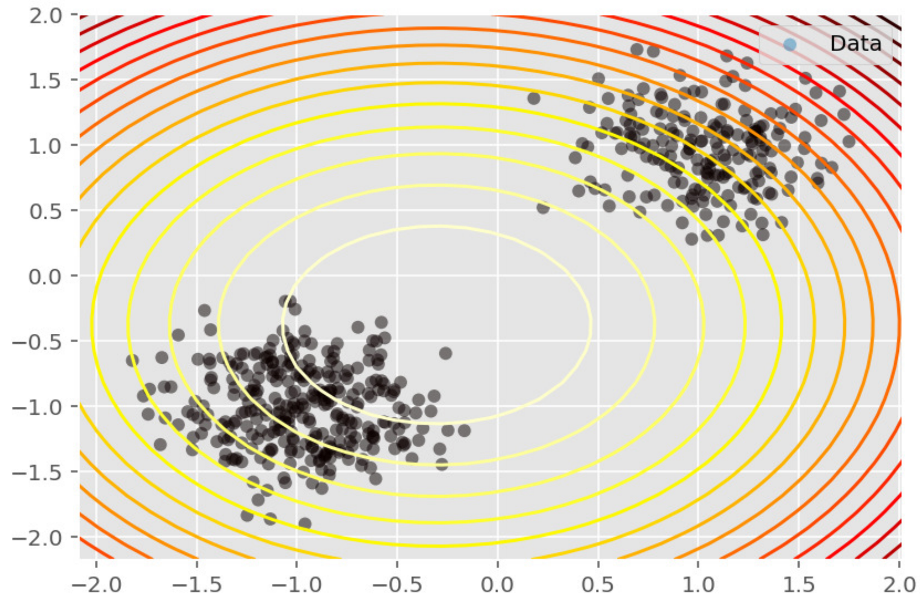
```
cov_latent = tau * pm.gp.cov.ExpQuad(2, 1°)
cov_noise  = sigma2 * tt.eye(n)
Cov_func   = cov_latent(X) + cov_noise)()
```



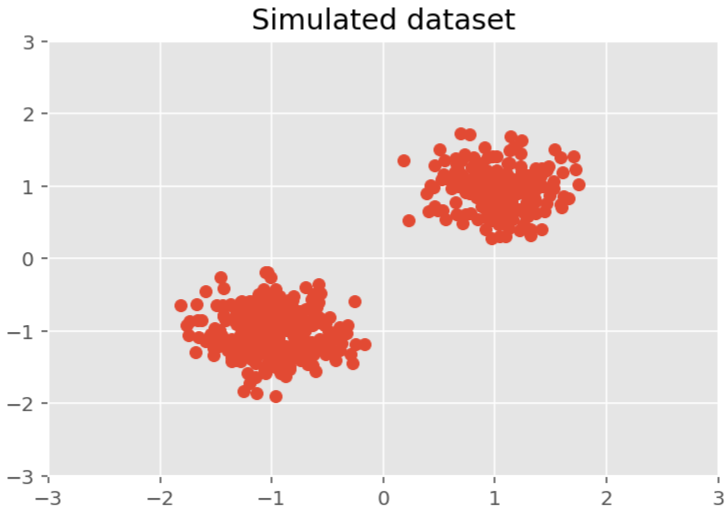
GP regression on fixation data

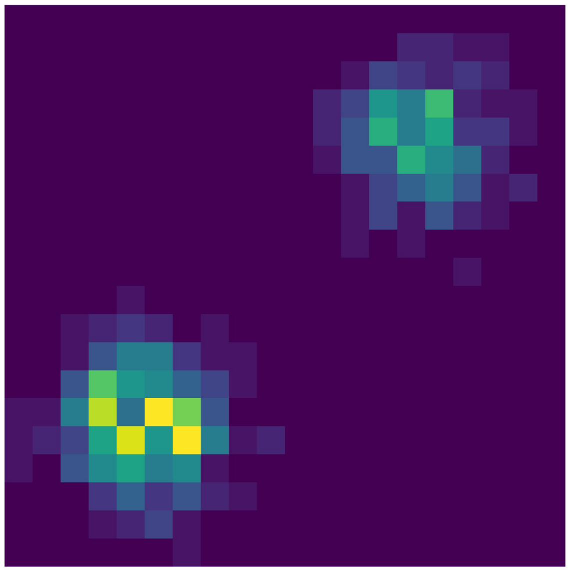


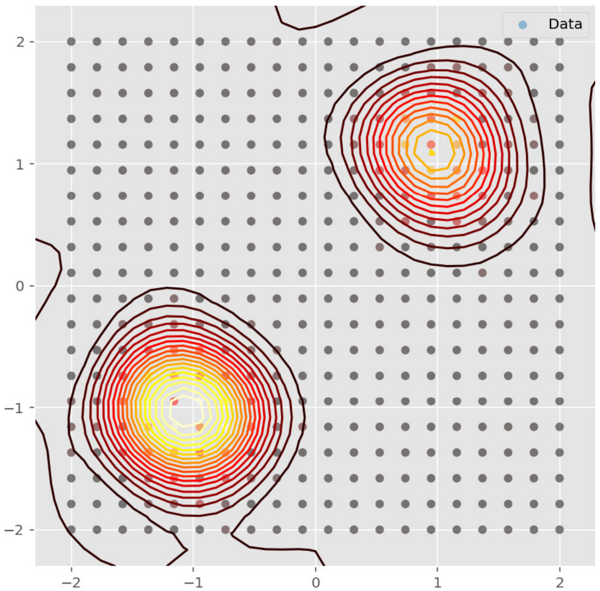
GP regression on sparse data



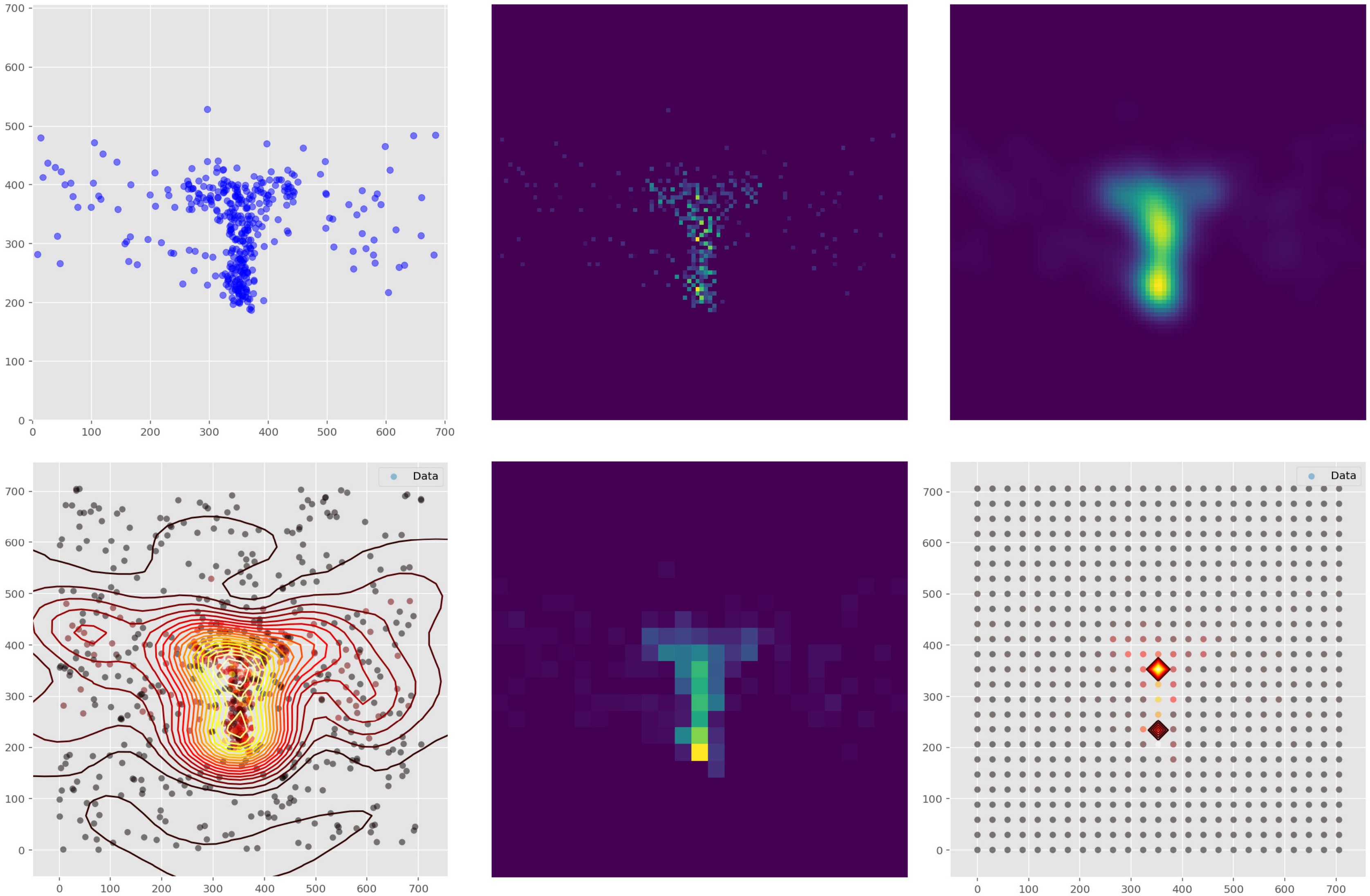
GP regression on sparse data





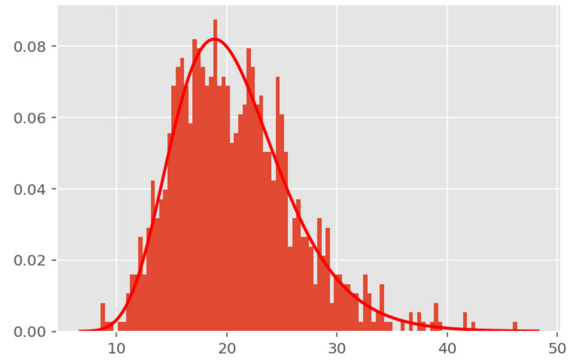


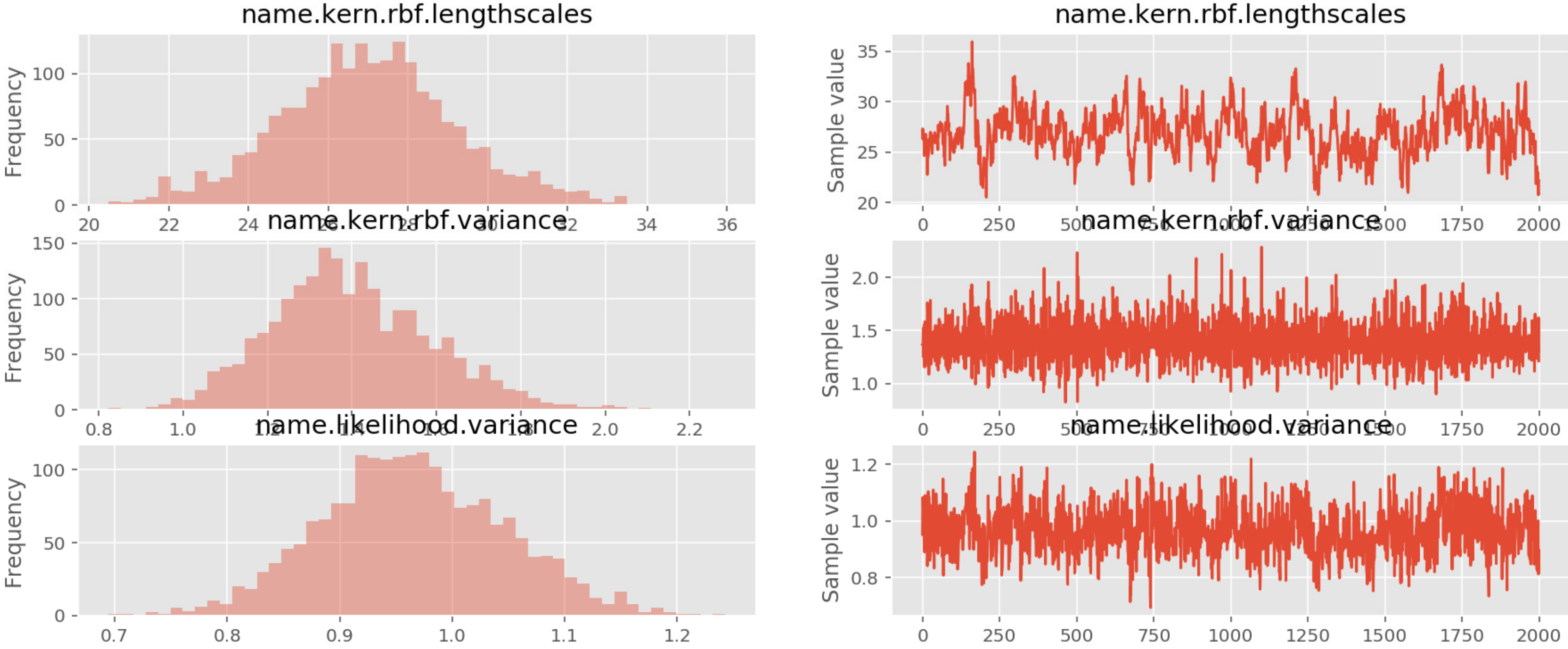
GP regression on fixation data



GP regression on fixation data

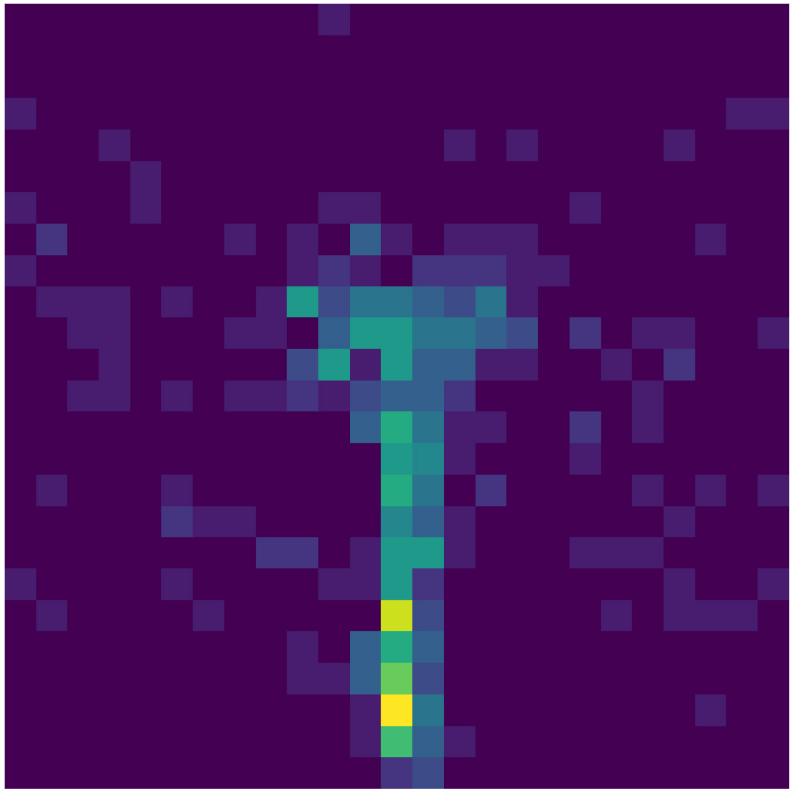
Informative Prior on length scale: ls ~ LogGaussian (mu = 1°, sd)

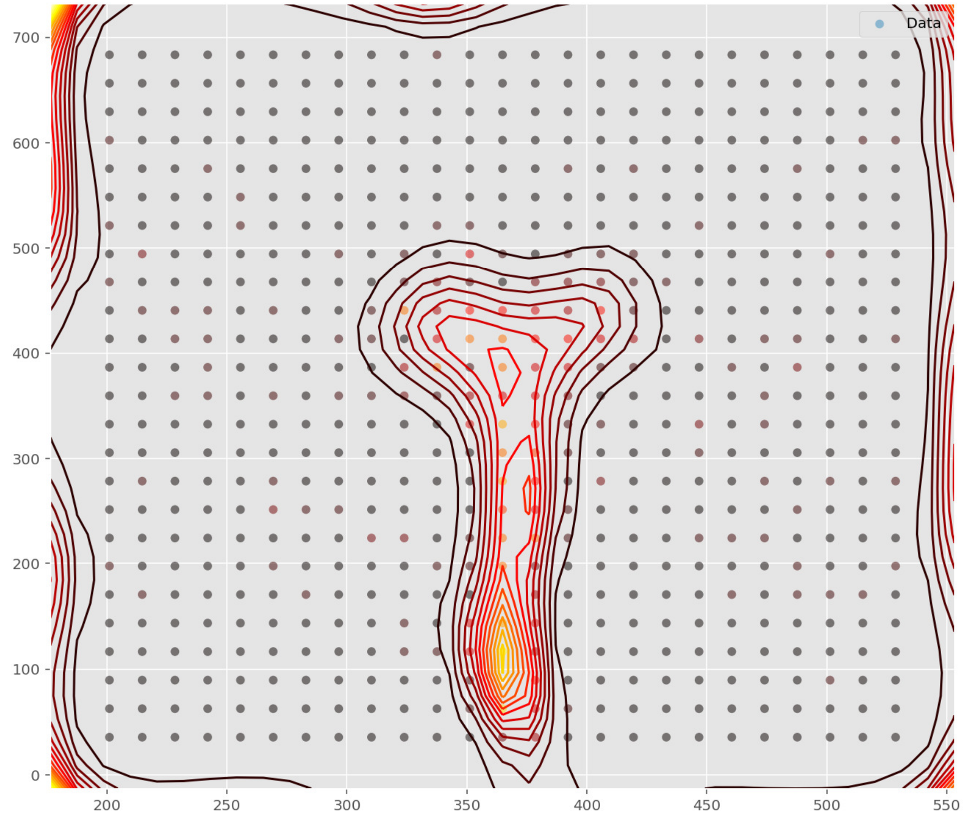




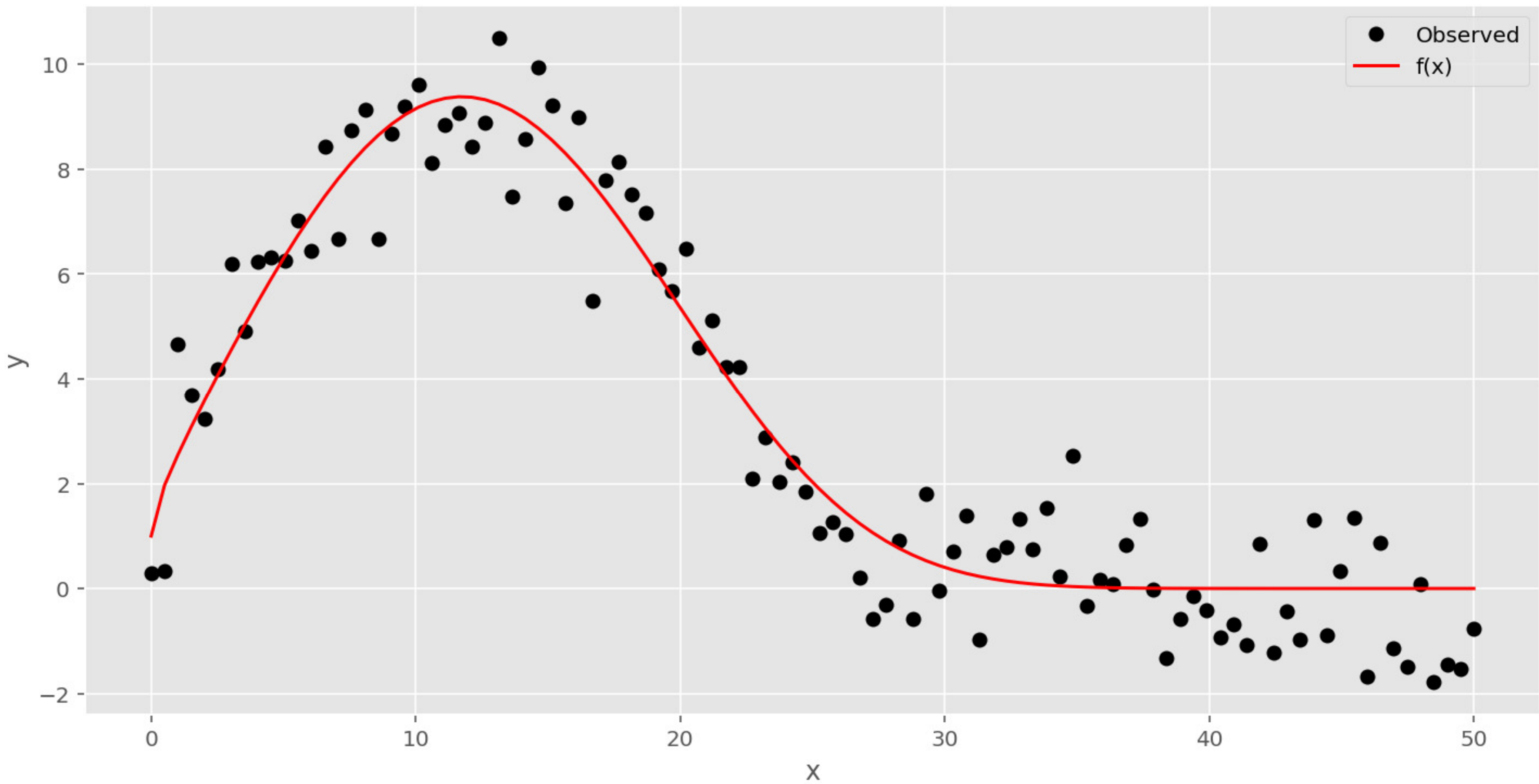
GP regression on fixation data

Informative Prior on length scale: ls ~ LogGaussian (mu = 1°, sd)





Overthinking:
what is smoothing, really?



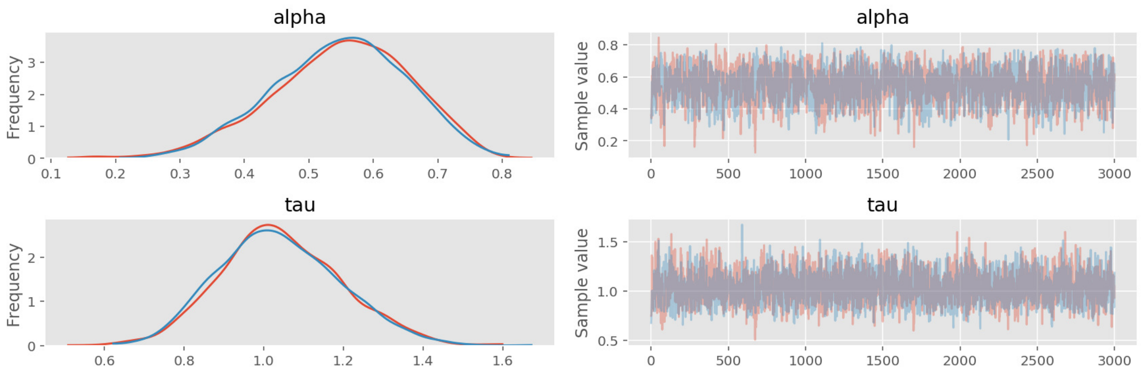
Overthinking: what is smoothing, really?

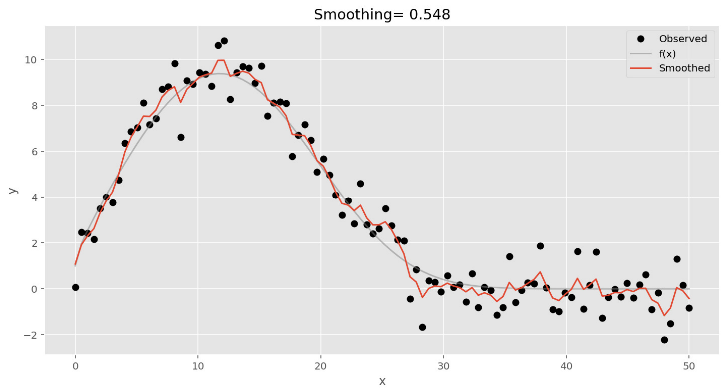
$$
\begin{aligned}
z_1 &\sim ImproperFlat(-\infty, \infty) \\
z_i &\sim \mathcal{N}(z_{i-1} + \mu, \sigma_1^2) \text{ for } i = 2, \ldots, N \\
y_i &\sim \mathcal{N}(z_i, \sigma_2^2)
\end{aligned}
$$

$$
\begin{aligned}
z_1 &\sim ImproperFlat(-\infty, \infty) \\
z_i &\sim \mathcal{N}(z_{i-1} + \mu, (1 - \alpha) \cdot \sigma^2) \text{ for } i = 2, \ldots, N \\
y_i &\sim \mathcal{N}(z_i, \alpha \cdot \sigma^2)
\end{aligned}
$$

$$z_1 \sim ImproperFlat(-\infty, \infty)$$
$$z_i \sim \mathcal{N}(z_{i-1} + \mu, (1-\alpha)\cdot\sigma^2) \text{ for } i = 2, \ldots, N$$
$$y_i \sim \mathcal{N}(z_i, \alpha \cdot \sigma^2)$$

```
with pm.Model() as model2:
    smth_parm = pm.Uniform('alpha', lower=0, upper=1)
    mu2 = pm.Normal("mu", sd=LARGE_NUMBER)
    tau2 = pm.Exponential("tau", 1.0/LARGE_NUMBER)
    z2 = GaussianRandomWalk("z",
                            mu=mu2,
                            tau=tau2 / (1.0 - smth_parm),
                            shape=y.shape)
    obs = pm.Normal("obs",
                    mu=z2,
                    tau=tau2 / smth_parm,
                    observed=y)
    trace = pm.sample(3e3, njobs=2, tune=1000)
```





Moving forward

- Scale up to more dimensions
  - Finer grid
  - Temporal information
- Hierarchical model
  - Subject effect, multiple condtions, etc.
- How wrong is it using Gaussian Process to model fixation data?
  - Dirichlet process?
- Unify model for both Fixation number and fixation duration:
  - Poisson Process for fixation number at location (x, y)
  - Gamma distribution for fixation duration (waiting time)

Thanks!

More information could be found on my Github repository:
junpenglao/Bayesian_Smoothing_EyeMovement

PyMC3 FNSNF iBMLAB Eye and Brain Mapping Laboratory UNI FR UNIVERSITÉ DE FRIBOURG UNIVERSITÄT FREIBURG

Bonus slide

Is it help to have mean function?