BAYES 20
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Using expert's knowledge in Bayesian analysis

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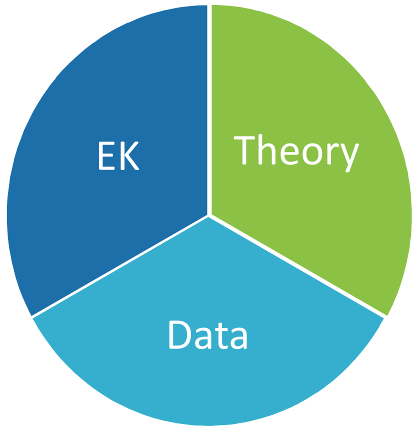
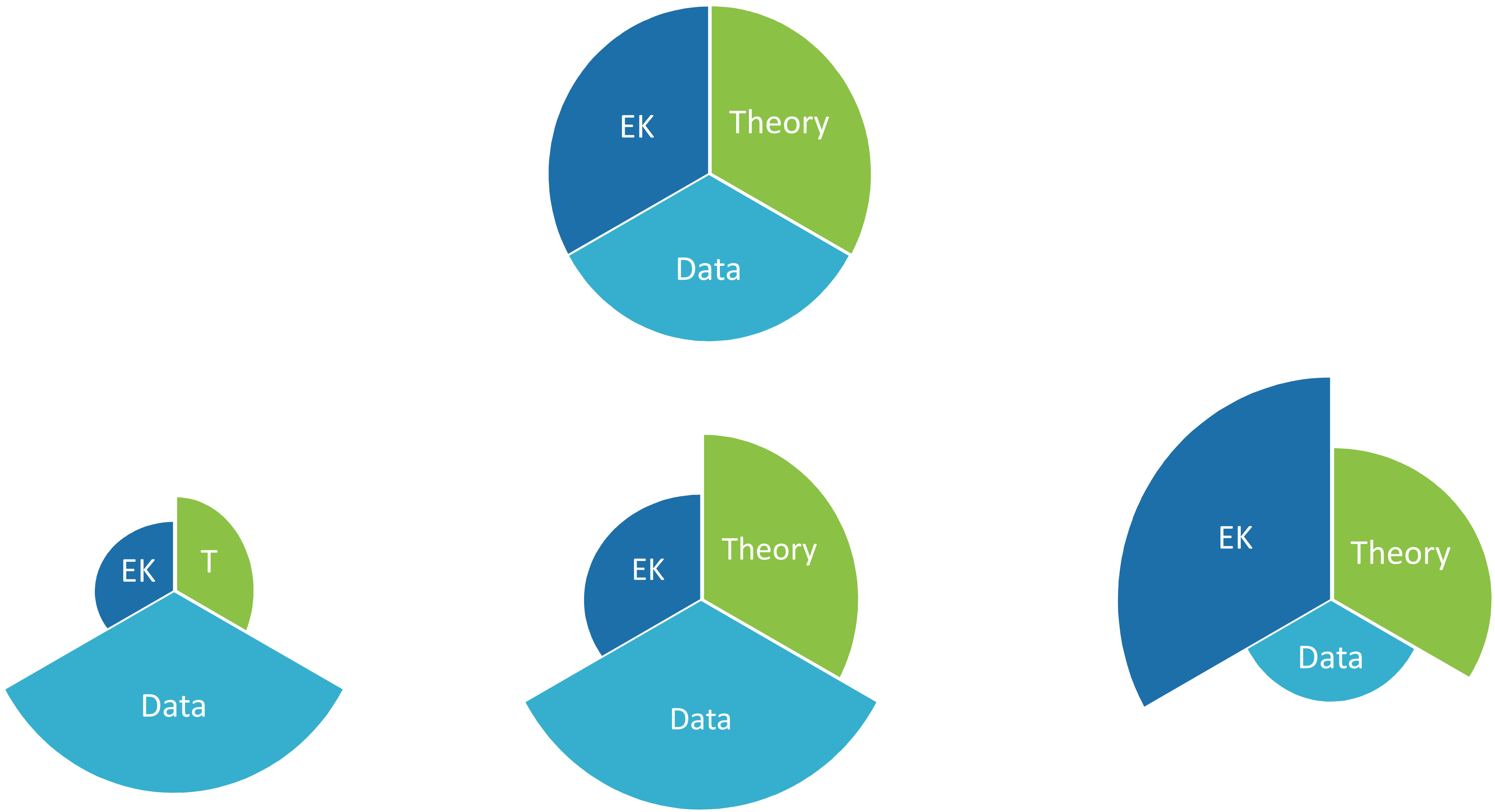

EK
Theory
Data


EK
Theory
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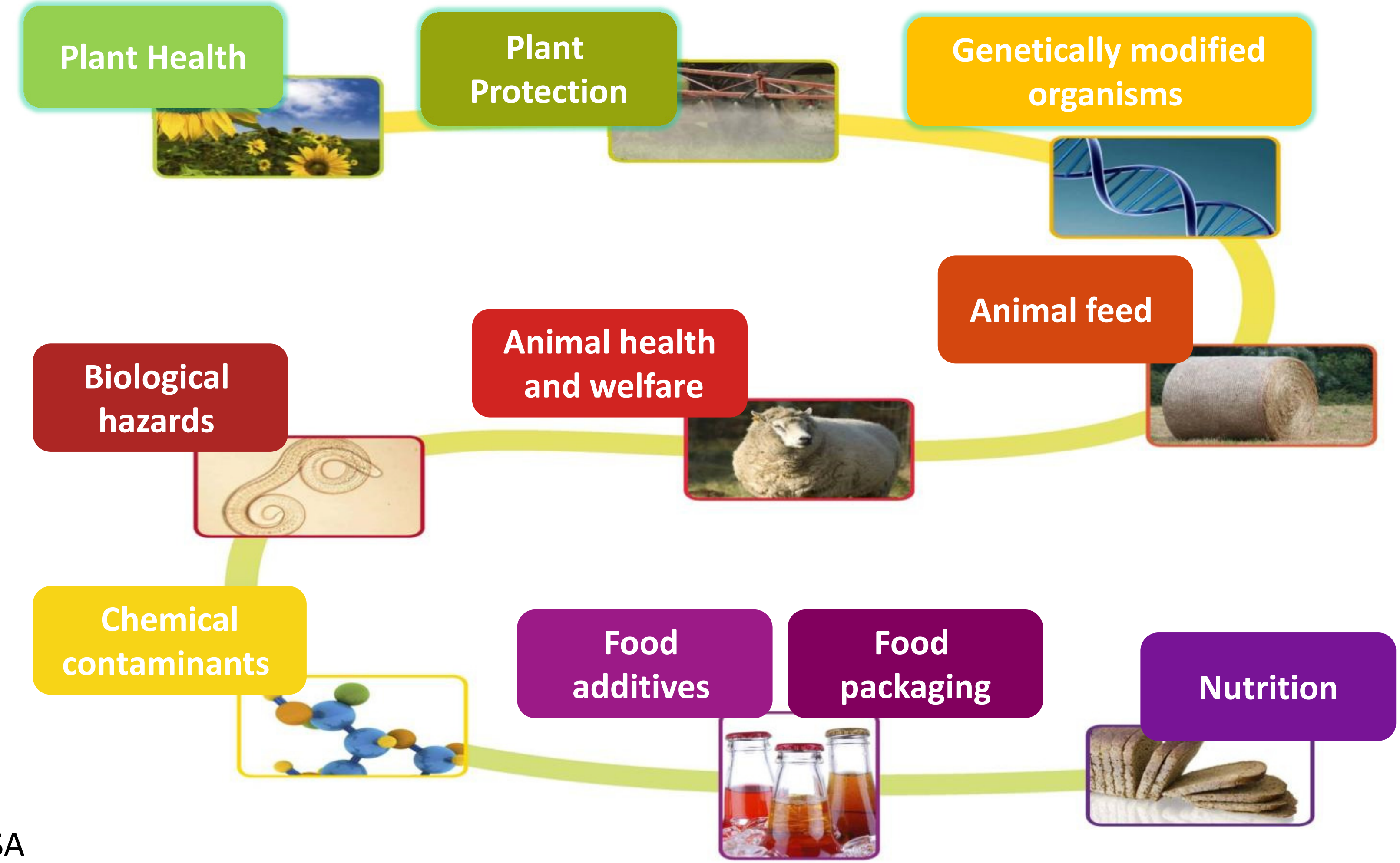
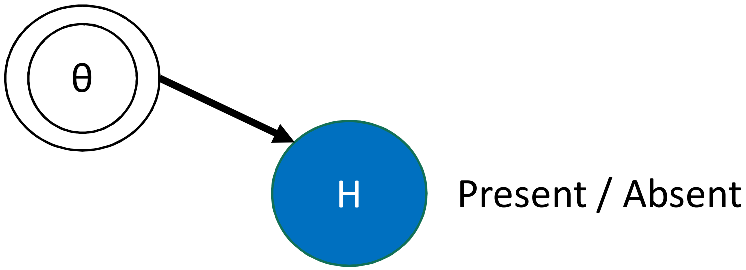
Theory
Data

European Food Safety Agency (EFSA)



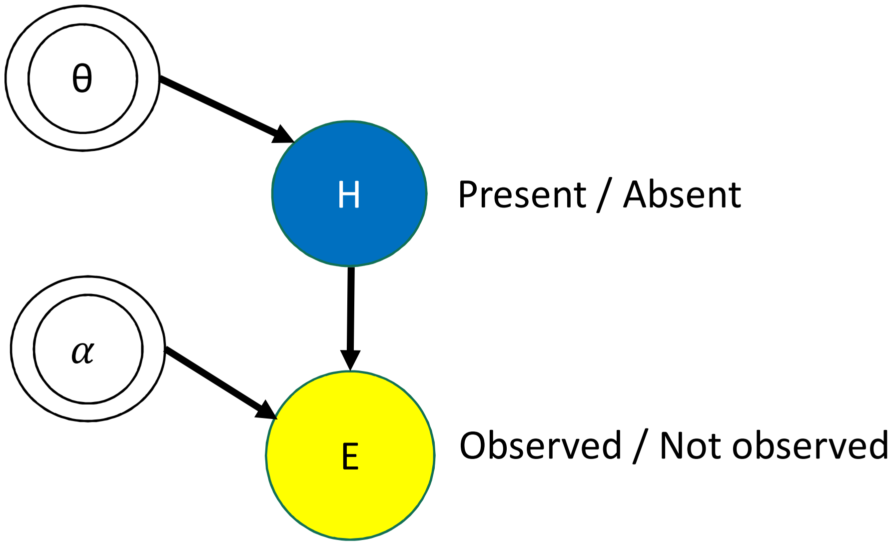
Source: EFSA

Is the crayfish still present?



H	
Present	θ
Absent	$1 - \theta$

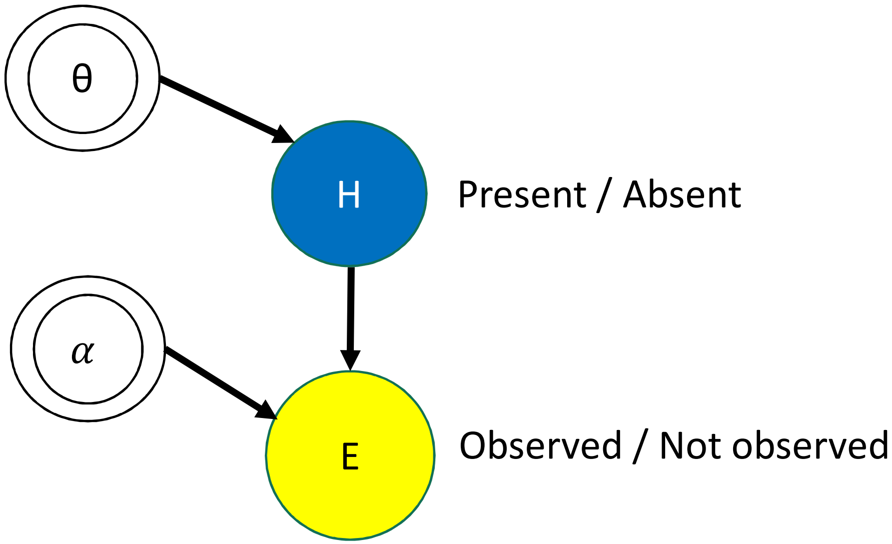
Is the crayfish still present?



H	
Present	θ
Absent	$1-\theta$

	H	
E	Present	Absent
Observed	α	0
Not observed	$1-\alpha$	1

Is the crayfish still present?

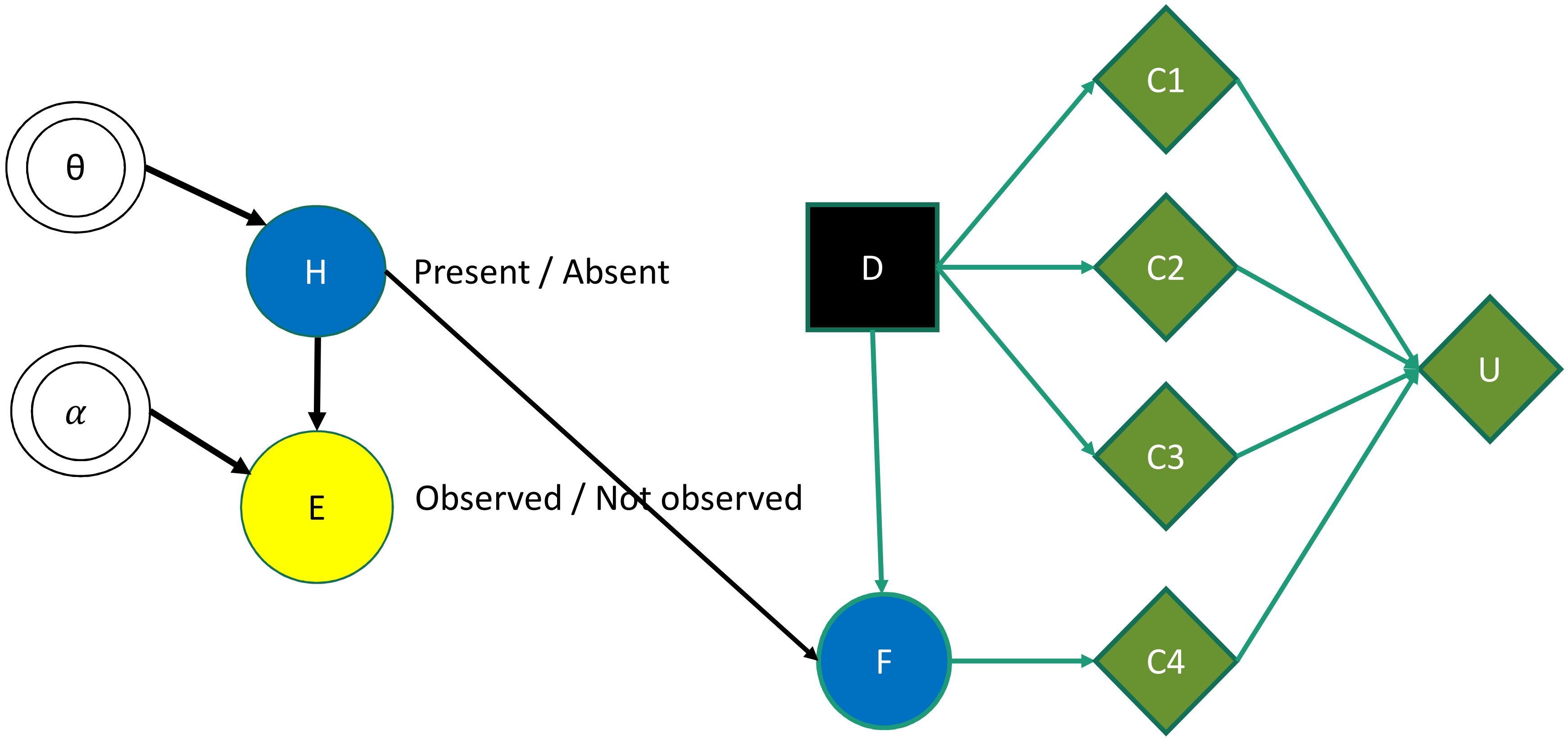


H	
Present	θ
Absent	1 − θ

	H	
E	Present	Absent
Observed	α	0
Not observed	$1-\alpha$	1

$$P(H|not\ E) = \frac{(1-\alpha)\theta}{1-\alpha\theta}$$

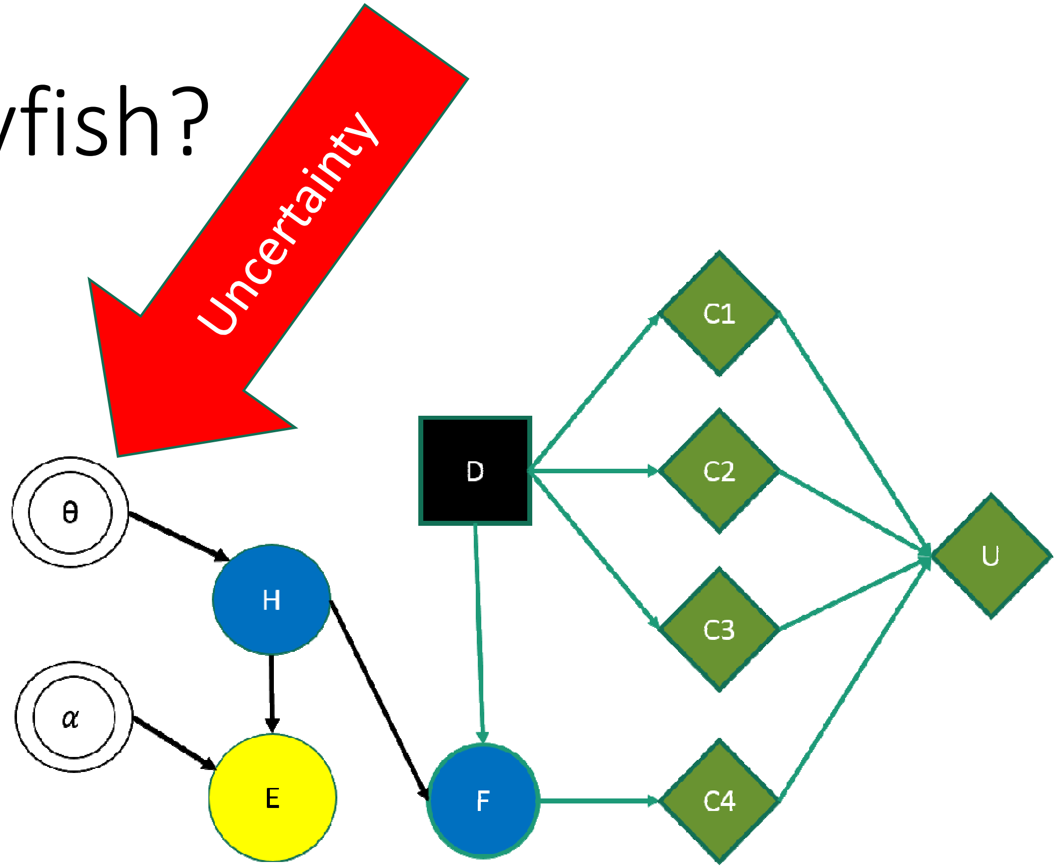
What to do with the crayfish?



What to do with the crayfish?

Management alternative	C1: Cost	C2: Neg Impact	C3: Accept-ance
Do nothing	*0*	*0*	*0*
Mechanical removal	*10*	*2*	*10*
Add poison	*5*	*10*	*2*

C4: Loss is even worse if crayfish is present after management



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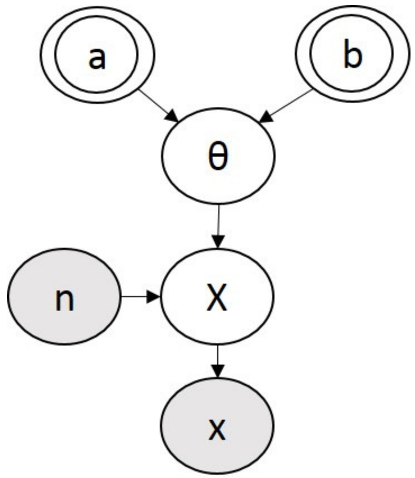
A simple Bayesian analysis



θ parameter of interest

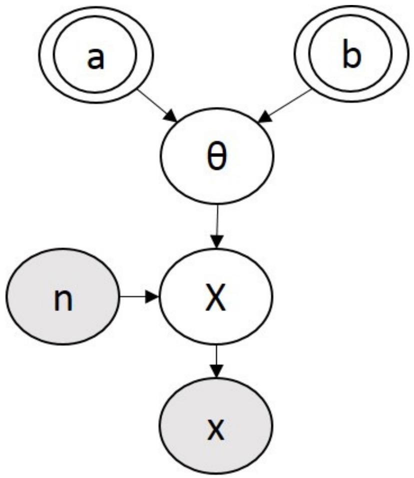
a & b expert knowledge on θ

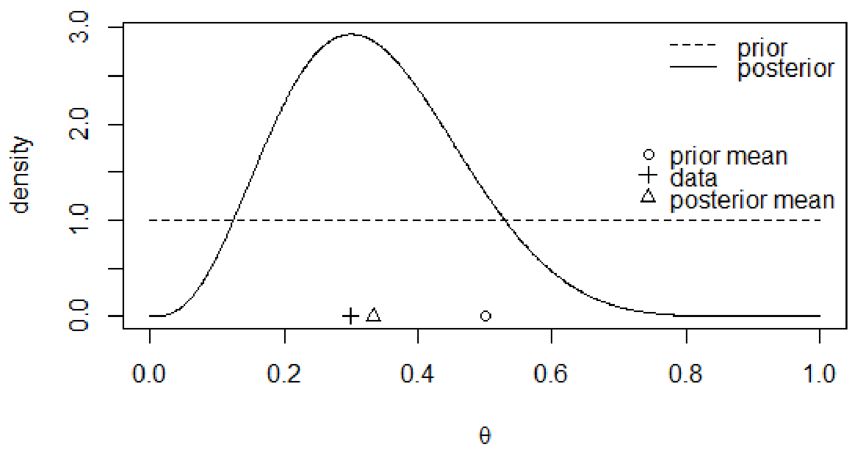
X system variable

n sample size

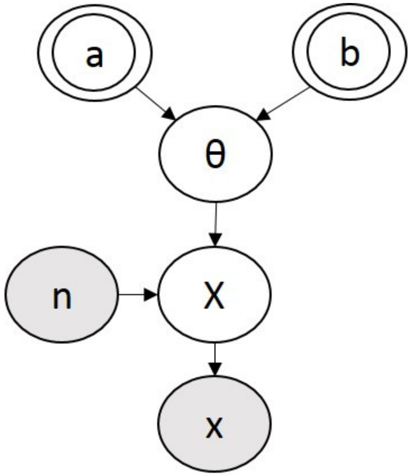
x observations of the variable

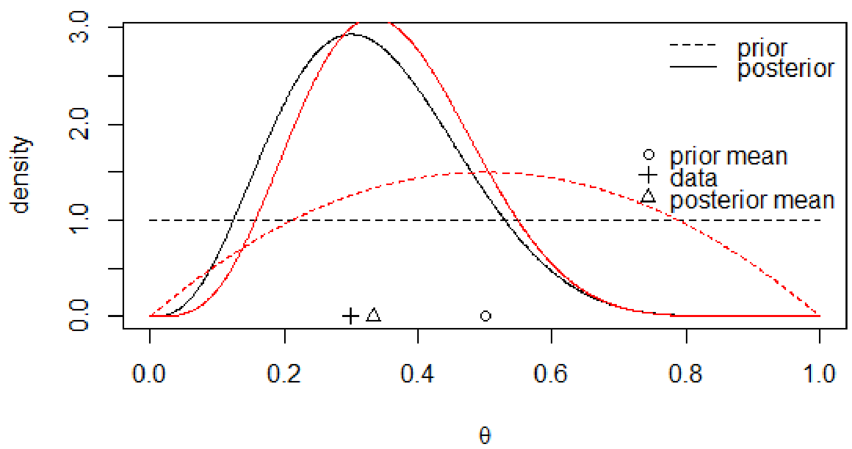
A simple Bayesian analysis



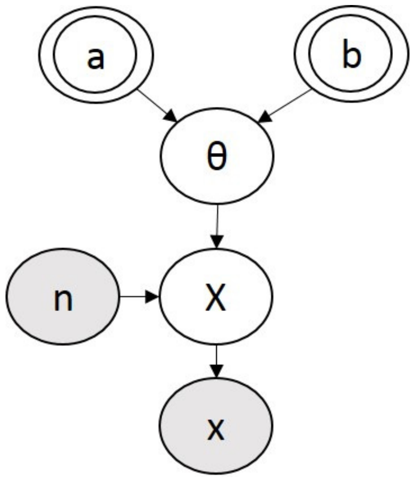


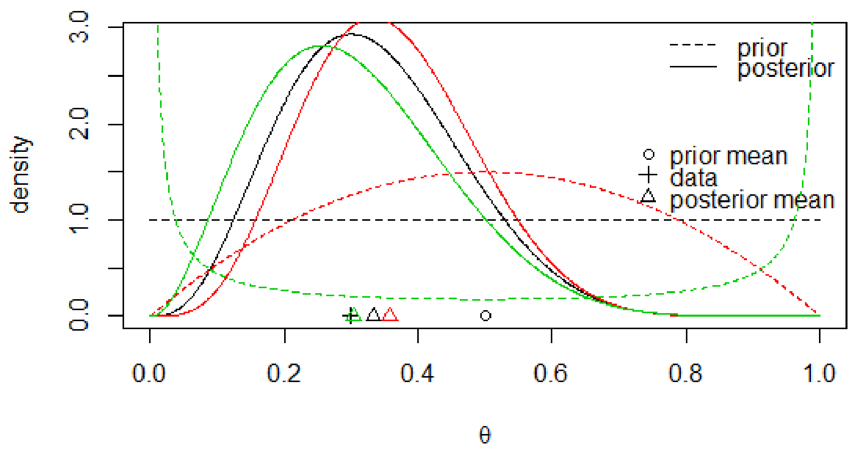
A simple Bayesian analysis





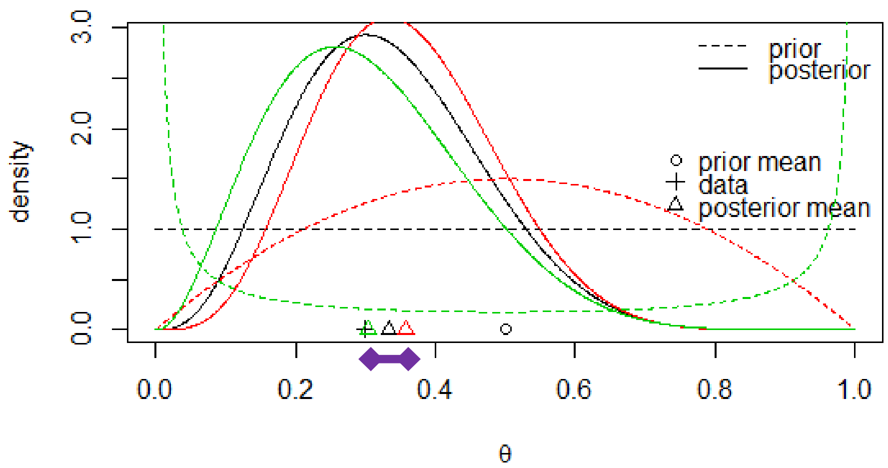
A simple Bayesian analysis



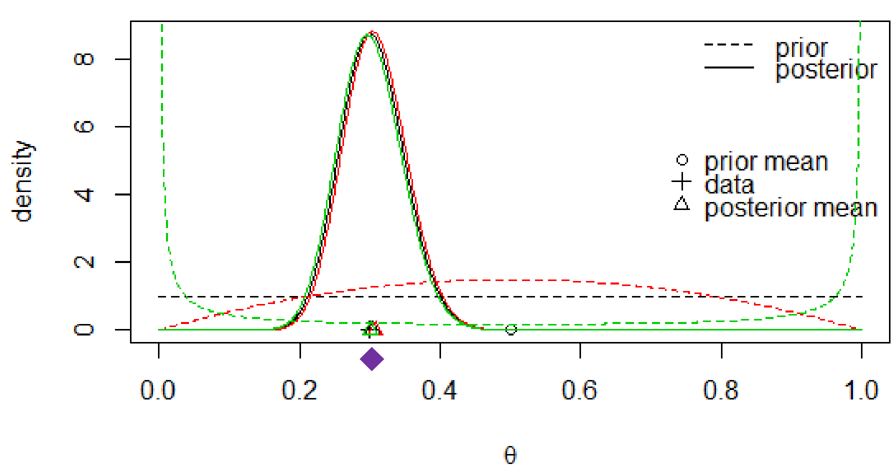


The prior may matter – better get it right

Small sample size



Large sample size



EK in risk and decision analysis

Quantitative risk models should be informed by systematically reviewed scientific evidence, however, in practice empirical evidence is often limited: in such cases it is necessary to turn to expert judgement.

Psychological research has shown that unaided expert judgement of the quantities required for risk modelling - and particularly the uncertainty associated with such judgements - is often biased, thus limiting its value.

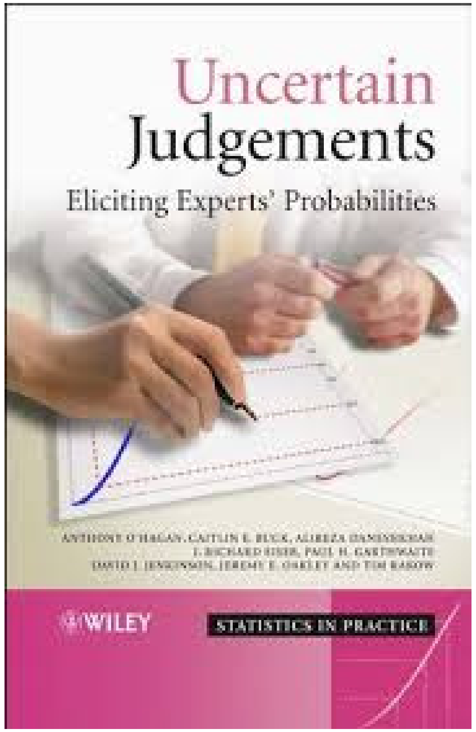
Accordingly methods have been developed for eliciting knowledge from experts in as unbiased a manner as possible.

https://www.efsa.europa.eu/en/press/news/140623

Source: EFSA

Expert's Knowledge Elicitation

- Aim to describe the Expert's Knowledge about one or more uncertain quantities in probabilistic form
- i.e. a joint probability distribution for the random variable in question
- EKE can be used to build priors distributions or prior predictive distributions



An Expert Knowledge Elicitation

- Formulate the elicitation questions
- Ask experts about
  - Probabilities
  - Quantiles
  - Probability intervals
  - Moments or other descriptions of a probability distribution
- Fit and aggregate into a probability distribution for the uncertain quantity

Direct methods for EKE

- Simple and a bit crude
  - *Intervals* – Lower and Upper limits, then a Uniform distribution
  - *Triangular distributions* – Mode, Lower and Upper limits
- Cumulative Density Function (CDF)
  - *Quartiles* – 4 intervals, median and 25th and 75th percentiles
  - *Tertiles* – 3 intervals with equal probability
  - *Probabilities/Hybrid* – Choose probabilites and intervals
- Probability Density Function (PDF)
  - Mode/Mean, percentiles, shape,…
  - Place chips, draw it by hand…

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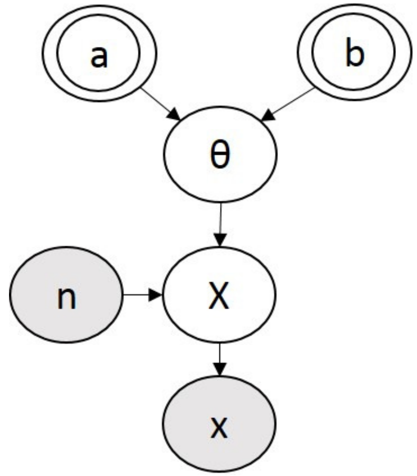
Indirect methods for EKE



- Equivalent Prior Sample (EPS)
  - *What is the expected frequency of the event?*
  - *What is the size a sample that you imagine to have behind this estimate?*

$$\frac{x}{n} = ? \qquad n = ?$$

- Hypothetical Future Sample (HFS)
  - *In a future sample of size 100 – in how many times has the event occured?*

$$n = 100 \qquad x = ?$$

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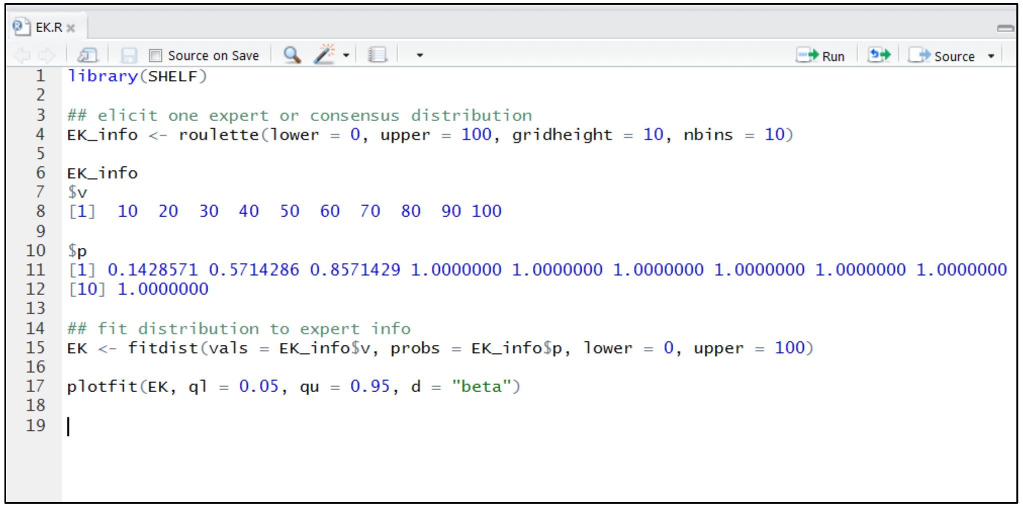
Selection of Structured EKE Software

- EXCALIBUR (EXpert CALIBration): www.lighttwist.net/wp/excalibur
- ElicitN: www.downloadcollection.com/elicitn.htm
- SHELF (The SHeffield ELicitation Framework): www.tonyohagan.co.uk/shelf/
- MATCH Uncertainty Elicitation Tool: optics.eee.nottingham.ac.uk/match/uncertainty.php#
- UncertWeb - The Elicitator: http://elicitator.uncertweb.org/
- Variogram elicitation: www.variogramelicitation.org
- Unicorn: www.lighttwist.net/wp/unicorn-download

Source: http://www.expertsinuncertainty.net/Software/tabid/4149/Default.aspx

An example
– elicit the probability of the crayfish individuals to survive the winter

- The SHELF R-package
- A web-interface for the SHELF R-package: optics.eee.nottingham.ac.uk/match/uncertainty.php#
- Roulette
- Quartiles
- Tertile


EK.R
Source on Save
Run
Source
library(SHELF)

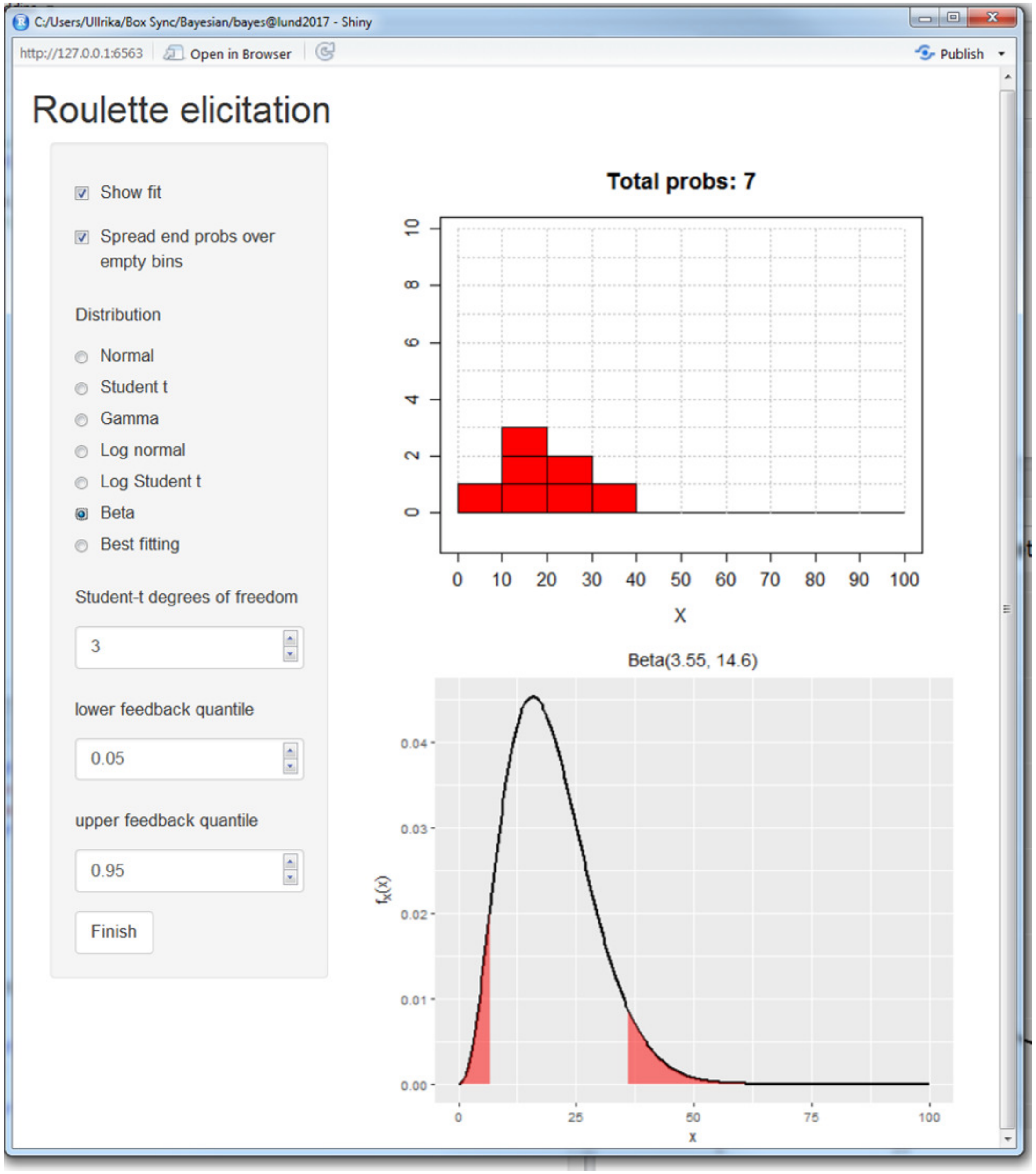
elicit one expert or consensus distribution
EK_info <- roulette(lower = 0, upper = 100, gridheight = 10, nbins = 10)

EK_info
$v
[1] 10 20 30 40 50 60 70 80 90 100

$p
[1] 0.1428571 0.5714286 0.8571429 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000 1.0000000
[10] 1.0000000

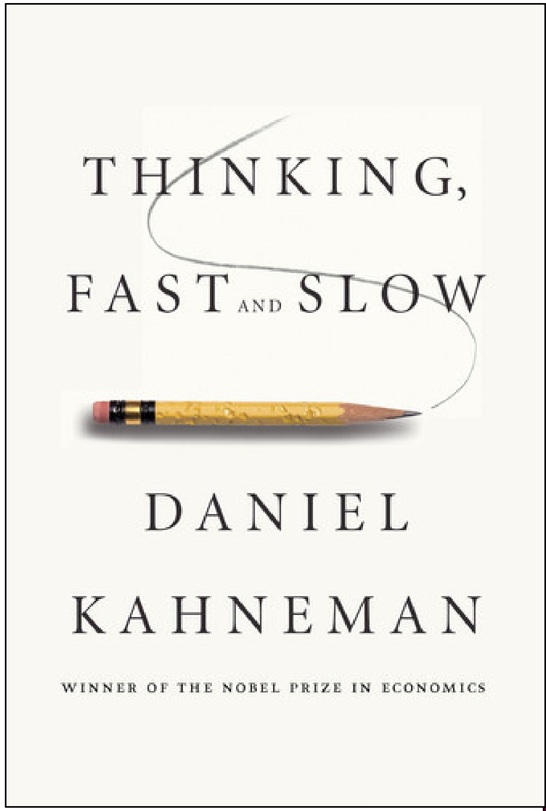
fit distribution to expert info
EK <- fitdist(vals = EK_info$v, probs = EK_info$p, lower = 0, upper = 100)

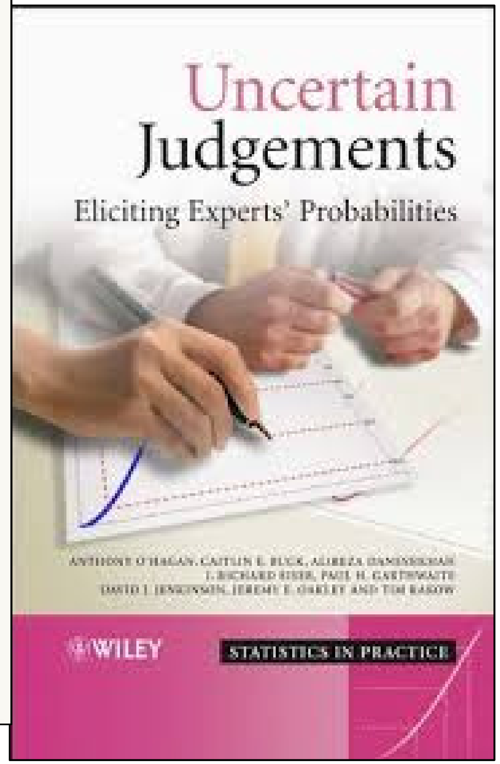
plotfit(EK, ql = 0.05, qu = 0.95, d = "beta")


C:/Users/Ullrika/Box Sync/Bayesian/bayes@lund2017 - Shiny
http://127.0.0.1:6563
Open in Browser
Publish
Roulette elicitation
Show fit
Spread end probs over empty bins
Distribution
Normal
Student t
Gamma
Log normal
Log Student t
Beta
Best fitting
Student-t degrees of freedom
3
lower feedback quantile
0.05
upper feedback quantile
0.95
Finish
Total probs: 7
0 10 20 30 40 50 60 70 80 90 100
X
Beta(3.55, 14.6)
0.00 0.01 0.02 0.03 0.04
0 25 50 75 100
x

Pshycological factors and elicitation

- Anchoring and adjustment
- Availability
- Range–frequency compromise
- Representativeness and baseline neglect
- Conjuction fallacy
- The law of small numbers
- Overconfidence





Elicitation with multiple experts

- Psychological factors when working with several experts

- Behavioural aggregation
  - Group elicitation
  - One or several iterations, individually and in group
- Mathematical aggregation
  - Treat each expert’s distribution as data and update the decision maker’s belief
  - Pooled opinions – linear or logarithmic pooling
  - Calibrate experts and weight according to their performance

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Alternative protocols for EKE

- the Sheffield protocol with group interaction of experts, consensus distributions
- the Cooke protocol with use of seed questions for the calibration of experts, no interaction
- a Delphi protocol on written expert elicitation with feedback loops, anonymous sharing of the results between iterations

An example
– elicit the probability of the crayfish individuals to survive the winter

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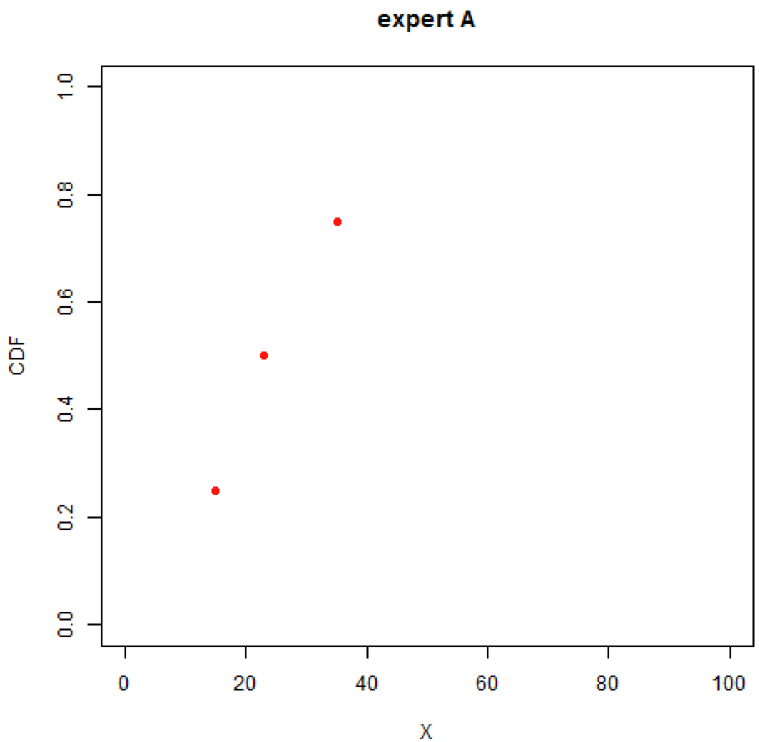
– elicit the probability of the crayfish individuals to survive the winter

75% percentile

Median

25% percentile

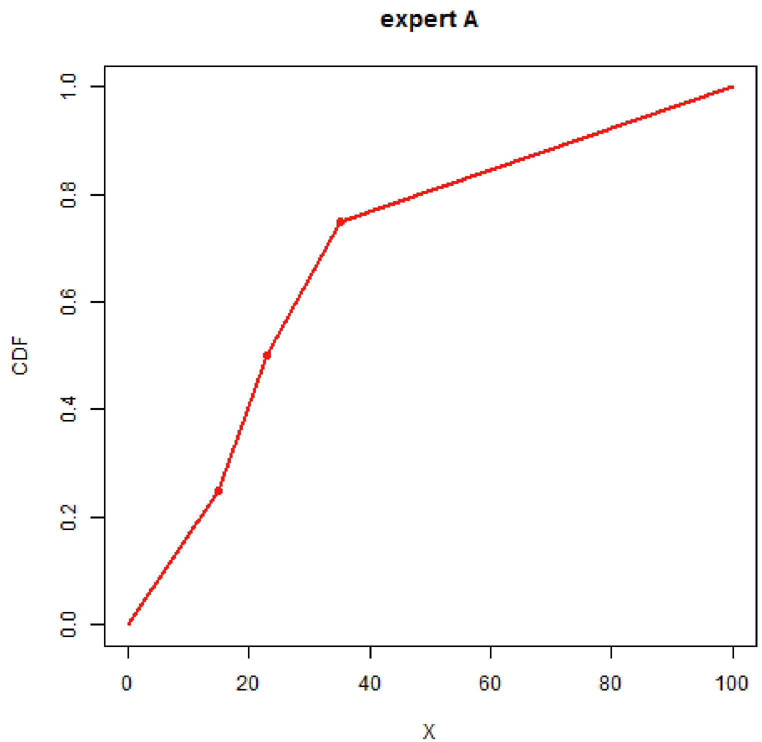


An example
– elicit the probability of the crayfish individuals to survive the winter

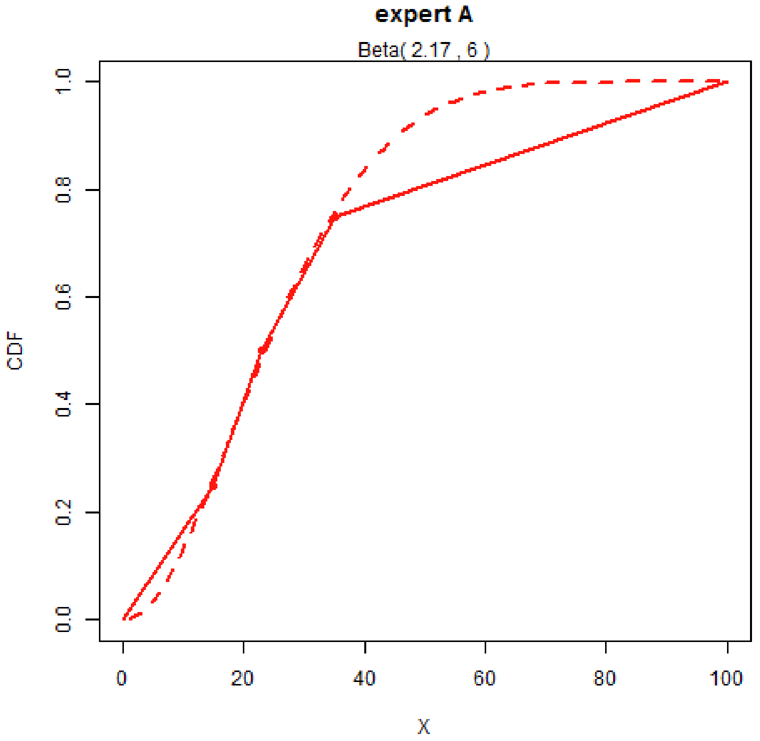
75% percentile

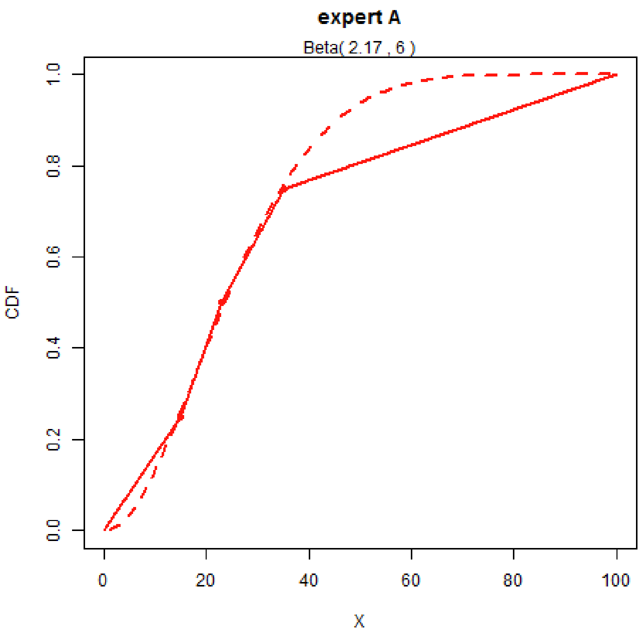
Median

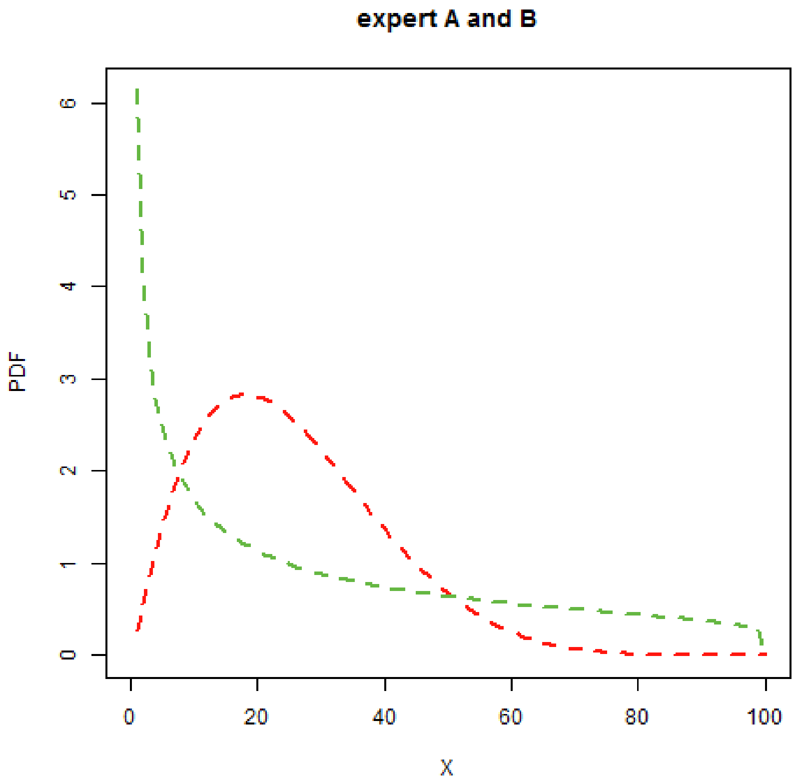
25% percentile

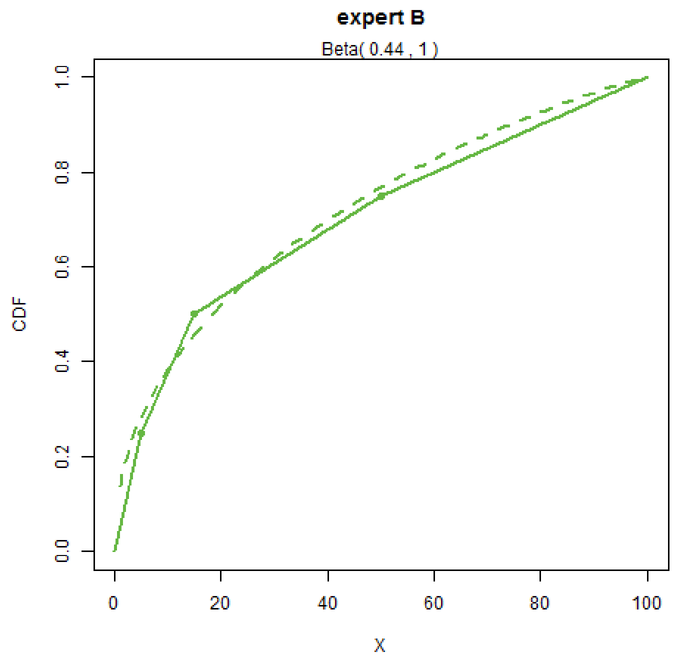


An example
– elicit the probability of the crayfish individuals to survive the winter









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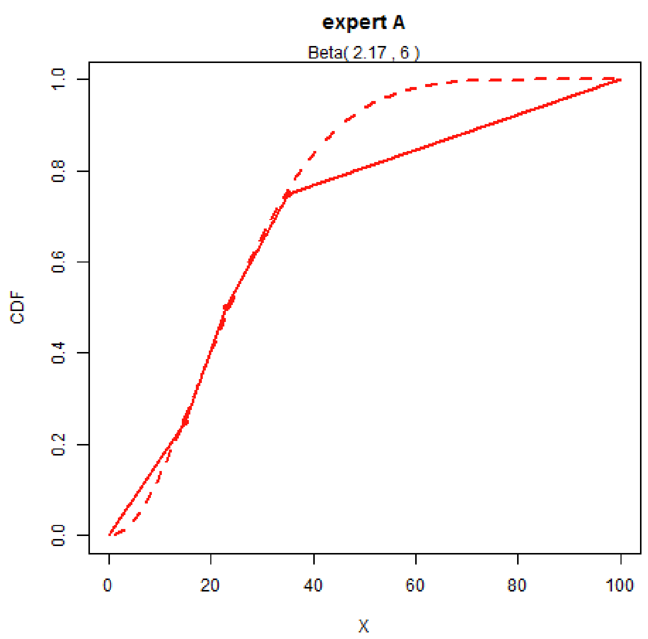

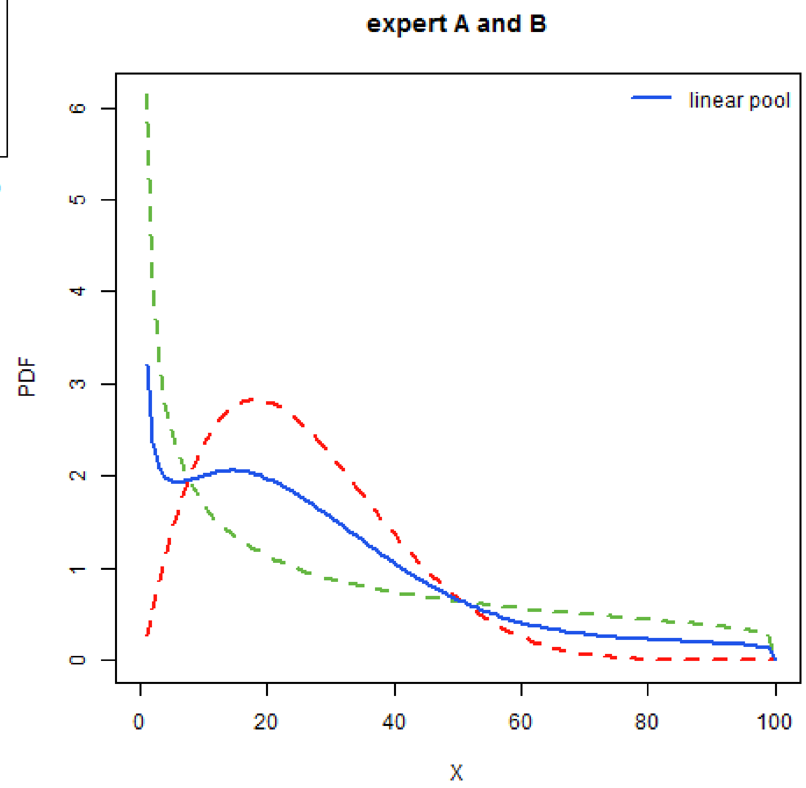

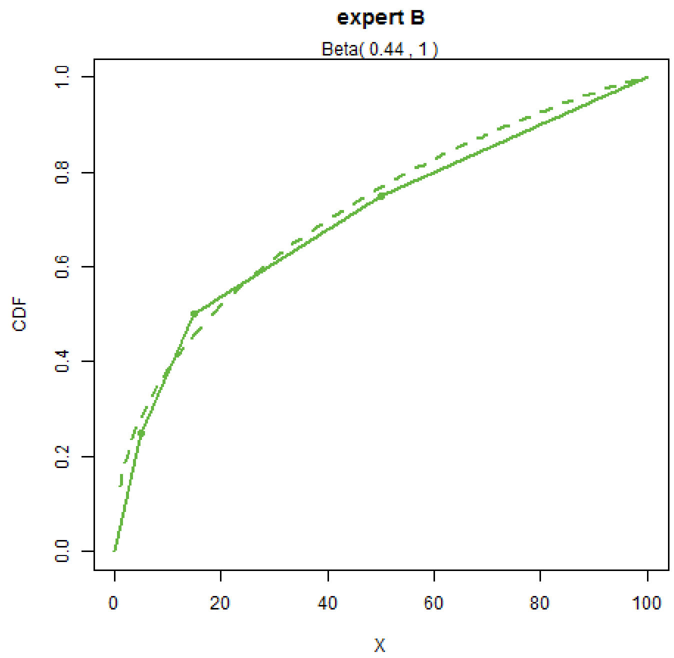

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Train the experts in making probabilistic judgments

- Get them custom to use probability density functions as a device for representing subjective uncertainty
- Clarify what is needed for the assessment, what are the uncertain quantities and how they are used to assess risk
- Reassure the experts understand that they will not be expected to claim certainty they do not have
- Encourage experts to be honest
- Give the experts a practice elicitation exercise
- Discuss psychological biases

dutiosb.twi.tudelft.nl/cost-delft/index.html

The state of the art in expert judgment

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The benefit of quantifying uncertainty using probability

- X is the proportion surviving the first spray
- Y is the proportion surviving the second spray
- Proportion surviving both applications is XY

X is small and Y is small, what is then XY?

What does ”small” mean?

The benefit of quantifying uncertainty using probability

- X is the proportion surviving the first spray
- Y is the proportion surviving the second spray
- Proportion surviving both applications is XY

X and Y without uncertainty result in a single value of of XY

A false sense of security

The benefit of quantifying uncertainty using probability

- X is the proportion surviving the first spray
- Y is the proportion surviving the second spray
- Proportion surviving both applications is XY

Using point estimates in input can result in biased estimate of overall risk

Plug in estimates – no uncertainty	Consider uncertainty in inputs
E(X) = 2%	X ~ U(0,4%)
E(Y) = 2%	Y ~ U(0,4%)
XY = E(X)E(Y) = 0.04%	E(XY) = 0.053

Aleatory and epistemic uncertainty

Uncertainty in Scientific Assessment

EFSA Journal

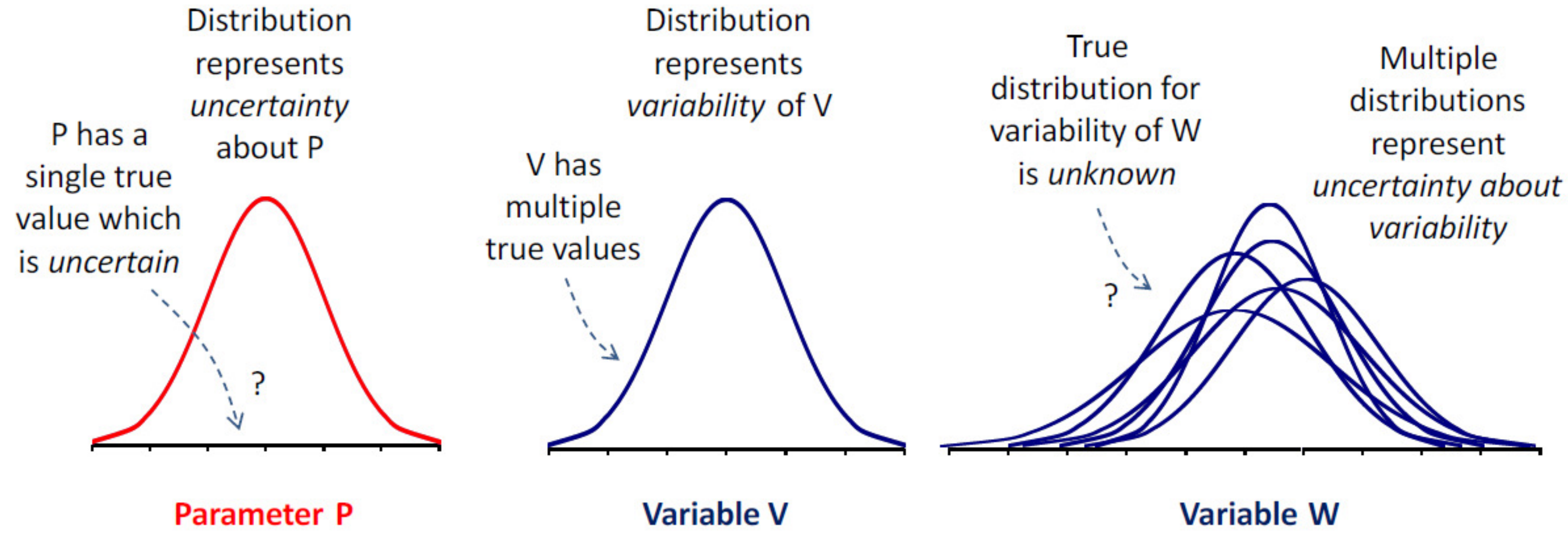

Figure 2: Illustration of the distinction between uncertainty and variability (left and central graphs), and that both can affect the same quantity (right hand graph).

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