SCHATTEN-VON NEUMANN PROPERTIES OF WEYL OPERATORS OF HÖRMANDER TYPE

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Let $t \in \mathbf{R}$ be fixed and consider the *pseudo-differential operators* $\operatorname{Op}_t(a)$ with *symbol* a which is defined by the formula:

$$\operatorname{Op}_t(a)f(x) \equiv (2\pi)^{-n} \iint_{\mathbf{R}^n \times \mathbf{R}^n} a((1-t)x+ty,\xi)f(y)e^{i\langle x-y,\xi\rangle}\, dyd\xi$$

It is well-known that if $0 \le \delta < \rho \le 1$ and $r \in \mathbf{R}$, then each $\operatorname{Op}_t(a)$ with $a \in S^r_{\rho,\delta}(\mathbf{R}^{2n})$ is L^2-continuous, if and only if $S^r_{\rho,\delta} \subseteq L^\infty$ (i.e. $r \le 0$). Here $S^r_{\rho,\delta}(\mathbf{R}^{2n})$ consists of all $a \in C^\infty(\mathbf{R}^{2n})$ such that

$$|\partial_x^\alpha \partial_\xi^\beta a(x,\xi)| \le C_{\alpha,\beta}(1+|\xi|)^{r-\rho|\beta|+\delta|\alpha|}.$$

More recently, results which are focused on "individual symbols" instead of whole symbol classes can be found in e.g. [1], from which it follows that if $a \in S^r_{\rho,\delta}$ for some r, then $\operatorname{Op}_t(a)$ is L^2-continuous, if and only if $a \in L^\infty$.

The general theory involving these results, is formulated within the Hörmander-Weyl calculus, where the symbol classes $S(m,g)$ are parameterized with weight functions m and Riemannian metrics g. The continuity investigations also involve Schatten properties. Especially, the following general result is deduced: Let $p \in [1,\infty]$, and assume that the g-Planck's function h_g satisfies $h_g^N m \in L^p$, for some $N \ge 0$. Then $\operatorname{Op}_t(a)$ is a Schatten-p operator, if and only if $a \in L^p$.

Recently, a related result was obtained also when $p \le 1$. More precisely, in [2] it is proved that if $p \in (0,1]$, $m \in L^p$ and $a \in S(m,g)$, then $\operatorname{Op}_t(a)$ is a Schatten-von Neumann operator of order p.

In the talk we explain these results with explicit examples, and present some ideas of some proofs.

[1] E. Buzano, J. Toft *Schatten-von Neumann properties in the Weyl calculus*, J. Funct. Anal. **259** (2010), 3080–3114.

[2] J. Toft *Continuity and compactness for pseudo-differential operators with symbols in quasi-Banach spaces or Hörmander classes*, arXiv:1406.3820 (2014).

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