THE POROUS MEDIUM EQUATION ON MANIFOLDS WITH CONICAL SINGULARITIES

ELMAR SCHROHE

ABSTRACT. We study the porous medium equation

$$\begin{aligned} \dot{u}(t) - \Delta(u^m(t)) &= f(u,t), \quad t \in (0, T_0], \\ u(0) &= u_0 \end{aligned}$$

on a manifold with conical singularities. We assume that $m > 0$ and $f = f(\lambda, t)$ is a holomorphic function of λ on a neighborhood of $\mathrm{Ran}(u_0)$ with values in Lipschitz functions in t on $[0, T_0]$.

We model the manifold with conical singularities by a compact manifold $\mathbb{B}$ with boundary of dimension ≥ 2, endowed with a degenerate Riemannian metric g, which, in a collar neighborhood $[0,1) \times \partial\mathbb{B}$ of the boundary $\partial\mathbb{B}$, is of the form $g(x, y) = dx^2 + x^2 h(y)$ for a (non-degenerate) Riemannian metric h on $\partial\mathbb{B}$. The Laplacian Δ associated with this metric naturally acts on scales of weighted Mellin-Sobolev spaces.

Given a strictly positive initial value u_0 we show existence, uniqueness and maximal L^p-regularity of a short time solution. In particular, we obtain information on the short time asymptotics of the solution near the conical point. Our method is based on bounded imaginary powers results and R-sectoriality perturbation techniques.

Actually, these solutions turn out to be instantaneously smooth in space for positive time outside the conic singularity. Moreover, we obtain long time existence of maximal regularity solutions when the initial data are positive and the forcing term f is zero and show how to establish the existence of weak solutions for the case of non-negative initial data and zero forcing term.

(Joint work with Nikolaos Roidos)

INSTITUT FÜR ANALYSIS, LEIBNIZ UNIVERSITÄT HANNOVER, WELFENGARTEN 1, 30167 HANNOVER, GERMANY

E-mail address: schrohe@math.uni-hannover.de

Date: May 12, 2016.

2010 *Mathematics Subject Classification.* 35J70; 35K59; 58J40.