Exercises: Least squares and numerical integration

Numerical Analysis, FMNF10, 2018

Least squares method

1. We would like to fit a straight line $y(x) = c_0 + c_1 x$ to the data below by using the least squares method.

x	-1	1	2
y	1	2	3

a) Derive the overdetermined system $Ac \approx b$, which the coefficients $c = (c_0, c_1)^{\mathrm{T}}$ need to fulfill.

b) Approximate a solution to the overdetermined system by the least squares method.

c) Plot the data points together with your straight line. Is the line a good fit to the data?

2. Consider the experimental measurements

x	0.00	1.00	2.00	3.00	4.00	5.00
y	0.03	0.15	0.89	2.79	6.42	12.5

a) Derive the overdetermined system that a polynomial of degree n, with $0 \leq n \leq 5$, needs to satisfy in order to fit the data points.

b) Write a code that approximates a solution to the overdetermined system by the least squares method for $n = 0, 1, \ldots, 5$.

c) Plot the data points together with the computed polynomials. Which polynomial seems to capture the general trend in the data?

Numerical integration

3. Consider the following MATLAB code:

```
N = 3 ;
a = 0 ;  b = 1 ;
dx = (b-a)/N ;
x = linspace(a+dx/2,b-dx/2,N)' ;
fx = cos(x) ;
weights = ones(1,N) ;
int = dx * weights * fx ;
```

a) Which numerical method is implemented in the code, and what does the quantity `int` approximate?

b) Modify the code (without changing the problem being solved) so that the numerical approximation is improved.

4 Consider the integral

$$\int_0^1 \sin(x^2)\,\mathrm{d}x.$$

a) Approximate the integral by employing the midpoint rule and dividing the interval $[0, 1]$ into $N = 3$ equally sized intervals.

b) How does the error of the midpoint rule decrease as the number of intervals N tends to infinity?

5 Consider the integral

$$\int_0^1 x^2(x+1)\,\mathrm{d}x.$$

a) Approximate the integral by employing Simpson's rule and dividing the interval $[0, 1]$ into $N = 2$ equally sized intervals.

b) Why is the approximation error equal to zero when applying Simpson's rule to the integral above?

6. Consider the approximation of the integral

$$\int_0^1 \frac{1}{1+25x^2}\,\mathrm{d}x = 0.274680\ldots$$

a) Compute numerically the integral above by employing the trapezoidal rule and dividing the interval $[0, 1]$ into $N = 4$ equally sized intervals.

b) Write a code for the trapezoidal approximation of the integral for a general N. How large do you need to choose N in order to obtain an answer with six correct digits?