Time Series Analysis
Fall 2017
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Prediction

The optimal k-step predictor of the ARMA process $A(z)y_t = C(z)e_t$ is obtained as

$$\hat{y}_{t+k} = \frac{G(z)}{C(z)}\epsilon_t$$

where $C(z) = A(z)F(z) + z^{-k}G(z)$. Here,

$\text{ord}(G) = \max(p - 1, q - k)$
$\text{ord}(F) = k - 1$

The prediction error is given as

$$\epsilon_{t+k|t} = y_{t+k} - \hat{y}_{t+k|t} = F(z)e_{t+k}$$

Note that this is an MA(k-1) process!

Matlab example of 5-step prediction of

$$(1 - 0.2z^{-1})\nabla_1 y_t = (1 - 0.3z^{-12})e_t$$

Code:

```matlab
s = 12;
k = 5;
C = [ 1 zeros(1,11) -0.3 ];
A = conv( [ 1 -0.2 ], [ 1 zeros(1,k-1) -1 ] );
[A,C] = equalLength( A, C );
[F, G] = deconv( conv( [ 1 zeros(1,k-1) ], C ), A );
```

Modeling the temperature in Svedala - first model:

$$A(z) = 1 - 1.79z^{-1} + 0.84z^{-2}$$
$$C(z) = 1 - 0.18z^{-1} - 0.11z^{-2}$$

One-step prediction... Is the prediction residual white? It is deemed normal distributed.
Similarly, an ARMAX process $A(z)y_t = B(z)x_t + C(z)e_t$ can be predicted as

$$\hat{y}_{t+k} = F(z)E\{x_{t+k}|Y_t, X_t\} + \frac{G(z)}{C(z)}x_t + \frac{\hat{G}(z)}{C(z)}y_t$$

where

$$F(z)B(z) = C(z)F(z) + z^{-k}\hat{G}(z)$$

with

$$\text{ord}(\hat{G}) = \max\{\text{ord}(C) - 1, \text{ord}(FB) - k\}$$

Note that this is the same order rules as before, but for the new polynomials.

The ARMAX process can be predicted as

$${\hat{y}}_{t+k} = F(z)E{x_{t+k}|Y_t, X_t} + \frac{\hat{G}(z)}{C(z)}x_t + \frac{\hat{G}(z)}{C(z)}y_t$$

where

$$F(z)B(z) = C(z)F(z) + z^{-k}\hat{G}(z)$$

with

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Note that this is the same order rules as before, but for the new polynomials.