Identification

Characteristics for the autocorrelation functions:

<table>
<thead>
<tr>
<th>Model</th>
<th>ACF $\rho(k)$</th>
<th>PACF $\phi_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$AR(p)$</td>
<td>Damped exponential and/or sine functions</td>
<td>$\phi_k = 0$ for $k &gt; p$</td>
</tr>
<tr>
<td>$MA(q)$</td>
<td>$\rho(k) = 0$ for $k &gt; q$</td>
<td>Dominated by damped exponential and/or sine functions</td>
</tr>
<tr>
<td>$ARMA(p,q)$</td>
<td>Damped exponential and/or sine functions after lag $q - p$</td>
<td>Dominated by damped exponential and/or sine functions after lag $p - q$</td>
</tr>
</tbody>
</table>
Electricity consumption in Australia

\[ \nabla_{12} \nabla \log y_t = (1 - 0.71B)(1 - 0.67B^{12}) \epsilon_t \]

Oxidant levels in Los Angeles
Testing for a non-zero mean

\[ \nabla y_t = e_t \]

\[ \nabla y_t = e_t + 1 \]

Reject the hypothesis that \( \hat{m}_y = m_y \), with significance \( \alpha \), if

\[ N \left( \hat{\Delta}_y - m_y \right) \sim \mathcal{N} \left( \hat{\Delta}_y - m_y \right) > \chi^2_{N-1}(\alpha/2) \]

Use the provided function \texttt{testMean}.

Identifying a Box-Jenkins model

Input and output of the simulated process in Example 4.21.

Begin by modeling the input signal, \( x_t \).
There seems to be strong dependencies for order 2 and 6, as well as, perhaps, at 4. In order to have a simple model, we begin with trying
\[ A_3(z) = 1 + a_2 z^{-2} + a_6 z^{-6} \]
We estimate the parameters of this model and examine the ACF and PACF of the residual (above). Looking at the ACF, there seems to be strong dependencies at lag 1 and 14. We thus modify our model to
\[ A_3(z) = 1 + a_2 z^{-2} + a_6 z^{-6} \\
C_3(z) = 1 + c_{14} z^{-14} \]
We re-estimating all parameters and examine the residuals. These are now deemed to be white.

We form the "white" input signal and the corresponding output
\[ w_t = A_3(z) x_t + \epsilon_t \]
and then estimate the transfer function from \( w_t \) to \( \epsilon_t \) as
\[ h_k = \frac{\sigma^2}{\sigma^2 \rho_{k,w}}(k) \]
The delay suggests \( d = 3 \). The impulse response seems to "ring", so we try \( r = 2 \). There seems to be 4 dominant components, i.e., \( s = 3 \).

We pretend that the additive noise is white, and estimate the parameters detailing the model
\[ y_t = B(z) z^{-d} x_t + \epsilon_t \]
where \( B(z) \) and \( A_2(z) \) are of order \( s \) and \( r \), respectively. We then compute the ACF and PACF of the residual \( \epsilon_t \) (above).

We form a model of the residual, beginning with using just \( a_1 \) and \( a_3 \). Examining the resulting residual suggest that we also needs \( \epsilon_1 \). This yields a white residual. Finally, we re-estimate all coefficients.