# Overview of course content

The goal of the course is to provide a set of tools and results used in modern inference theory. Many of the key concepts are applied to modern survival analysis, density and regression function estimation, spectral density estimation, and thus the course can also be seen as an introduction to these areas. We will also cover some nonstandard topics, such as empirical process theory for stationary data, and estimation of the spectral measure of stationary processes. A brief sketch of the course content is:

- **Introduction** (Chapter 1). The introduction!

- **Weak convergence** (Chapter 18). This is based on Hoffman-Jörgensen theory, and treats processes that are not necessarily measurable, meaning they are not "random" and not possible to calculate the probability of. One (very important) example is the ordinary empirical distribution function! This is IF we use the sigma-algebra generated by the open balls in the supnorm topology (which is what one would do). A highlight of the course is to show the elegant way out of this.

- **Empirical processes** (Chapter 19). Discusses functional versions of the Law of Large Number (LLN) and  and Central Limit Theorem (CLT), the so called Glivenko-Cantelli and Donsker theorems. This is however done in more generality than usual. Having these results makes nonparametric tests of equality of distributions between two (or several) random variables (such as Kolmogorov-Smirnov tests)  a piece of cake.

- **Functional differentiability** (Chapter 20). Discusses how properties of smooth functionals of the distribution function, such as the mean and the variance, is used to get LLN and CLT for estimators of such. This immediately gives you consistency and asymptotic normality of the estimator, which is very convenient. It gives you the theoretical justification for the so called "plug-in" method.

- **Quantiles and order restrictions** (Chapter 21). Uses the empirical process theory to derive asymptotic normality of nonparametric estimators of quantiles.

- ***L-statistics** (Chapter 22)

- **Bootstrap** (Chapter 23). Derives a functional CLT for the bootstrap, and uses functional differentiability to prove that the bootstrap works, in many situations.

- **Nonparametric density and regression function estimation** (Chapter 24). A density function is not (!) a smooth functional of the distribution function and therefore the above results can not be applied to density estimators. We discuss kernel estimators and order restricted estimators of a density. We also discuss the analog problems for estimating a regression function.

- ***Empirical and partial sum process for long range dependent data**. Not all data are independent. We discuss the cases when data come from a stationary process, and treat the above problems (some of them) in those instances. Both weekly dependent and long range dependant data are covered.

- ***The empirical spectral process**. Discusses nonparametric estimators of the spectral density such as the periodogram and recent results on order restricted estimators.

- **M and Z-estimators** (Chapter 5). The Maximum Likelihhod estimator and the Least Squares estimator are the most commonly used estimators. They are in fact special cases of M and Z-estimators. We use the above results to derive (very easily) asymptotic normality for these.

- ***Other applications** (Chapter 25). For instance treating the Cox model in survival analysis, or other semiparametric models.

The *-marked parts will be covered partially/optionally, depending on the interest of the participants and on how much time we have.