Questions for Oral Exam on course
MASC02 Inference Theory

1 Point estimation

1. State the point estimation problem, using loss functions and risk functions. Show that it is not possible to obtain a uniformly best estimator. What approaches are there to solve that problem?

2. Define group families and exponential families of distributions. Give two examples on each.

3. Assume $X$ has an exponential family distribution. Derive the moment generating function, and show how this can be used to get moments.

4. What is a sufficient statistic, and how does randomization enter? Assume $\delta$ is an estimator based on $X \sim P_\theta$ and $\delta'$ an estimator based on a sufficient statistic $T$: Relate the risk of $\delta(X)$ to that of $\delta'(T)$.

5. Derive the sufficient statistic in a sample of size two for a Poisson distribution.

6. State and prove a necessary and sufficient condition for a statistic to be sufficient in a finite family of distributions.

7. State and prove Basu’s theorem.


9. Assume $T$ is a complete sufficient statistic for $\mathcal{P} = \{P_\theta : \theta \in \Omega\}$. Let $g(\theta)$ be U-estimable, and $L(\theta, d)$ a loss function convex in $d$. Show that there is an unbiased estimator that uniformly minimizes the risk in the set of unbiased estimators. What is this estimator called when the loss is quadratic?

10. In which situation does one need inverse binomial sampling? Derive the distribution for the relevant r.v.

11. Let $\mathcal{P}$ be a family of distribution and $\mathcal{G}$ a group of transformations of $\mathcal{P}$. What is the principle of equivariance?
12. Give necessary and sufficient conditions for an estimator to be MRE.

13. Derive the Pitman estimator of the parameter in a location family.

14. What is the setup for Bayesian inference? Assume quadratic loss function, and give an expression for the (a posteriori) Bayes estimator.

15. What is the relation between minimax estimators, Bayes estimators and least favorable distributions?

16. Assume a Bayes estimator has constant risk. Is it minimax?

2 Decision theory

1. Let \( \delta \) be a decision function. What are the possible decisions (i.e. the range of \( \delta \)) for (i) point estimation, (ii) confidence intervals, (iii) hypothesis testing. How is an optimal decision defined? Do optimal decisions always exist?

2. What is the difference between invariant and equivariant decisions? In what situations do they occur?

3. Relate admissible decisions, complete and minimal complete classes of decisions.

4. State and prove the Neyman-Pearson lemma.

5. Characterize, with proof, the UMP test for families of distributions with monotone likelihood ratios.

6. For distributions with monotone likelihood ratios, state conditions on loss functions under which one obtains essentially complete classes of tests. (You do not need to prove minimality.)

Good Luck!