1) a) The pole-zero plot in $A$ has a zero at the unit circle at angle $\omega = 2\pi 0.2$ and a pole close to the unit circle at $\omega = 2\pi 0.3$. The corresponding spectral estimate should then have a zero at $f = 0.2$ and a peak at $f = 0.3$. It could possibly be figure 1 or 2, but figure 1 seems to rather have two zeros and no peak. So most likely is that $A$ belongs to 2. The pole-zero plot in $B$ has two zeros and is most likely to belong to figure 1, where the pole-zero plot in $C$, that has a pole at $\omega = 2\pi 0.2$ belong to the spectral estimate in figure 3, which has a peak at $f = 0.2$. Process $A-2$ is an ARMA(2,2), $B-1$ is an MA(4) and $C-3$ an AR(2).

b) The periodogram variance is approximately $R^2 X(f)$ and the reduction of the Welch method is assumed to be the number of possible windows. The variance is approximately $V[\hat{R}_x(f)] = R^2 X(f) \approx 0.5 \cdot 64 \cdot 7 + 32 = 256$.

2) The covariance function for an MA($q$)-process is,

$$r(\tau) = \sigma^2 \sum_{j-k=\tau} c_j c_k.$$ 

We get

$$r(\tau) = \begin{cases} 
2(1^2 + (-1)^2) = 4, & \tau = 0 \\
2(1 - 1) = 0, & |\tau| = 1 \\
2(1 - 1) = 0, & |\tau| = 2 \\
2(-1) = -2, & |\tau| = 3 \\
0, & |\tau| > 3.
\end{cases}$$ 

The spectral density is

$$R_X(f) = 2|1 - e^{i2\pi 3 f}|^2 = 4 - 4\cos(2\pi 3 f) \quad -0.5 < f \leq 0.5.$$ 

3) The expected value of the derivative is always $E[X'(t)] = 0$ and the spectral density is $R_X'(f) = (2\pi f)^2 R_X(f)$. The variance of the derivative is

$$V[X'(t)] = \int R_X'(f) df = c \int_{f_0}^{f_0} (2\pi f)^2 (1 - |f| / f_0) df = 2c \int_0^{f_0} (2\pi f)^2 (1 - f / f_0) df = 8\pi^2 c f_0^3 / 12 = 1.$$ 

4) a) The Yule-Walker-equations give

$$4 + 2a_1 - 0.5a_2 = \sigma^2$$
$$2 + 4a_1 + 2a_2 = 0$$
$$-0.5 + 2a_1 + 4a_2 = 0,$$

with the solution $a_1 = -0.75$, $a_2 = 0.5$ and $\sigma^2 = 2.25$. The spectral density function is

$$R_x(f) = \frac{2.25}{|1 - 0.75 e^{-i2\pi f} + 0.5 e^{-i4\pi f}|^2} = \frac{2.25}{1.8125 - 2.25 \cos(2\pi f) + \cos(4\pi f)}.$$
The output spectral density is \( R_w(f) = 1 \). The Wiener filter becomes

\[
H(f) = \frac{R_z(f)}{R_z(f) + R_w(f)} = \frac{2.25}{2.25 + |1 - 0.75e^{-12\pi f} + 0.5e^{-4\pi f}|^2}
\]

Thus, we obtain the following formula:

\[
H(f) = \frac{2.25}{4.0625 - 2.25\cos(2\pi f) + \cos(4\pi f)}.
\]

5) a) **Correct**: The frequencies after sampling are 1000 Hz (no aliasing), 5000 Hz, (aliasing of \( f_2 = 5000 \) Hz), and 17000 Hz (no aliasing, aliasing of \( f_3 = 17000 \) Hz).

b) **False**: The frequency function is

\[
H(f) = \begin{cases} 1 - \frac{1}{10000} \cdot |f| & |f| \leq 10000, \\ 0 & |f| > 10000. \end{cases}
\]

The remaining frequencies after the filtering are 1000 Hz and 5000 Hz where the frequency 17000 Hz is cancelled. The process \( Y(t) \) is sampled with \( f_s = 7000 \) Hz and the resulting frequencies are 1000 Hz (no aliasing), 5000 Hz (no aliasing) and 17000 Hz (aliasing of \( f_3 = 17000 \) Hz). The frequencies of the corresponding discrete time process \( Z_t \) are 10000/f = 1/20, 3000/f = 3/20 and 5000/f = 1/4. Two frequencies remain, 1/20 and 3/20, after filtering with the discrete time lowpass filter which passes all frequencies below \( |v| \leq 3/16 \).

c) **Correct**: The frequencies after sampling with \( f_s = 20000 \) Hz are 1000 Hz (no aliasing), 5000 Hz (no aliasing) and 17000 Hz (no aliasing). 3000 Hz (aliasing of \( f_3 = 3000 \) Hz). The frequencies of the corresponding discrete time process \( Z_t \) are 1000/f = 1/20, 3000/f = 3/20 and 5000/f = 1/4. Two frequencies remain, 1/20 and 3/20, after filtering with the discrete time lowpass filter which passes all frequencies below \( |v| \leq 3/16 \).

6) The output spectral density is

\[
R_Y(f) = \frac{R_X(f)}{|1 + 0.5e^{-12\pi f}|^2},
\]

with \( H(f) = \frac{1}{1 + 0.5e^{-12\pi f}} \). We can then write \( X_k = \sum_{u=0}^{\infty} (-0.5)^uX_{k-u} \). The cross-covariance is

\[
r_{X,Y}(\tau) = C[X_k, \sum_{u=0}^{\infty} (-0.5)^uX_{k-u+\tau}] = \sum_{u=0}^{\infty} (-0.5)^uC[X_k, X_{k-u+\tau}],
\]

yielding

\[
r_{X,Y}(\tau) = r_X(\tau) - 0.5r_X(\tau - 1) + 0.25r_X(\tau - 2) + \ldots.
\]

We obtain \( r_{X,Y}(\tau) = 0 \) for \( \tau \leq -2 \), \( r_{X,Y}(-1) = r_X(-1) = 0.2 \), \( r_{X,Y}(0) = r_X(0) - 0.5r_X(-1) = 1 - 0.1 = 0.9 \), and

\[
r_{X,Y}(\tau) = (-0.5)^{\tau+1}r_X(-1) + (-0.5)^\tau r_X(0) + (-0.5)^{\tau-1}r_X(1),
\]

when \( \tau \geq 1 \), implying

\[
r_{X,Y}(\tau) = (-0.5)^\tau (-0.9 + 1 - 0.4) = 0.5(-0.5)^\tau.
\]