Mathematical Statistics, Centre for Mathematical Sciences, Lund University
Exam in Stationary stochastic processes (FMSF10/MASC04)
Date: 2018-08-31
Time: 8.00 – 13.00
Help: Calculator, table of formulas in stationary stochastic processes, small table of formulas in mathematics.

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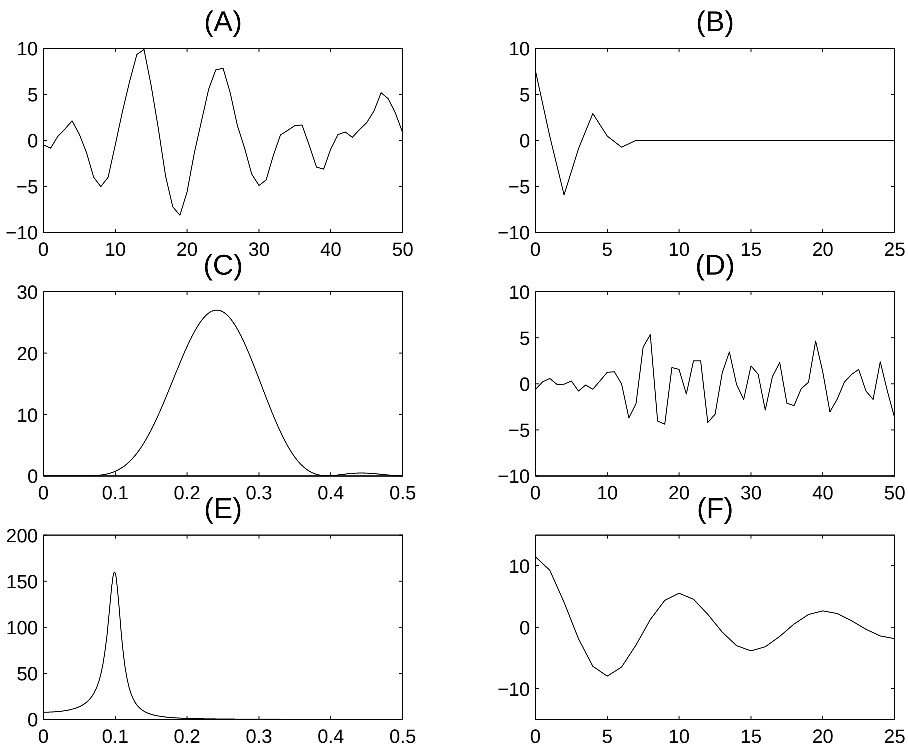
Solutions, correct and well motivated, of exercise 1-3 give 10 credits, and of exercise 4-6 give 20 credits. The maximal number of credits are 90. The limit for passing the course is 40 credits. Each exercise should have a solution starting on a new sheet of paper.

1) The following figures show different realizations, covariance functions, and spectral densities from one AR(2)-process and one MA(6)-process. Determine, with motivation, what figures that are realizations, covariance functions, and spectral densities. Also state and explain which realization, covariance function, and spectral density that are connected and which process that is the AR-process and which is the MA-process.

(10p)



2) A weakly stationary process $X(t)$, $t \in \mathbb{R}$, has the covariance function $r_X(\tau) = \frac{1}{1+\tau^2}$ and expected value $E[X(t)] = 0$. The process is filtered through an ideal bandpass filter,

$$H(f) = \begin{cases} 1 & 1 < |f| < 2, \\ 0 & \text{for all other values.} \end{cases}$$

Determine the output variance from the filter.

(10p)

3) A zero-mean weakly stationary process $X(t), t \in \mathbb{R}$ has the spectral density

$$R_X(f) = \begin{cases} 1.5 - |f| & \text{if } |f| \leq 1.5, \\ 0 & \text{for other values} \end{cases}$$

The process should be sampled with a certain sampling distance d.

a) What is the largest possible sampling distance to avoid aliasing?

(3p)

b) Determine the spectral density for the sampled process if the sampling distance is $d = 1$.

(7p)

4) A deterministic signal

$$s_t = \begin{cases} 5 & \text{for } t = 0, \\ 4 & \text{for } t = 1, \\ 3 & \text{for } t = 2, \\ 2 & \text{for } t = 3, \\ 1 & \text{for } t = 4, \end{cases}$$

is used in a digital communication system. Design the optimal matched filter to determine whether the signal has been sent or not, when one receives $s_t + N_t$, if the signal was sent, and N_t, if the signal was not sent. Determine the decision time and decision threshold for equal error rates, and compute the corresponding error probabilities. Assume N_t to be white Gaussian noise with $E[N_t] = 0$ and $V[N_t] = 1$.

(20p)

5) A stationary Gaussian process $X(t), t \in \mathbb{R}$, has expected value $E[X(t)] = 2$ and covariance function

$$r_X(\tau) = e^{-3\tau^2}.$$

The process $Y(t)$ is created by setting $Y(t) = 4X'(t)$. Determine the probability density

$$P(Y(t) < 5 + 0.5Y'(t)).$$

(20p)

6) The winter indoor temperature Y_t, $t = 0, \pm 1, \pm 2, \ldots$ (t=time in hours) could be assumed to correlate with the outdoor temperature X_s, $s \leq t$ where $Z_t = X_t + 3$ is defined as an AR(1)-process with covariance function $r_Z(\tau) = 2 \cdot 0.9^{|\tau|}$. The indoor and the outdoor temperatures are coupled according to

$$Y_t - 18 = 0.4X_t + 0.9X_{t-1} + e_t$$

where e_t, $t = 0, \pm 1, \pm 2, \ldots$, is zero-mean white Gaussian noise which is uncorrelated with X_t. Determine the cross-covariance function $r_{X,Y}(\tau)$ and the cross-spectrum $R_{X,Y}(f)$.

(20p)