1) The figures below belong to processes of type AR. Determine which pole-zero-plots, covariance functions and spectral densities that belong to the same process. Also, state which order each of the processes has.

![Pole-zero-plots](A,B,C)
![Covariance functions](I,II,III)
![Spectral densities](1,2,3)

(10p)

2) Let $\epsilon_t, t = 0, \pm 1, \pm 2 \ldots$, be discrete time uncorrelated Gaussian noise with expected value $m$ and variance $\sigma^2$. Define a new stochastic process $X_t$ by

$$X_t = 0.6\epsilon_t + 0.3\epsilon_{t-1} + 0.1\epsilon_{t-2}.$$ 

Calculate the expected value, covariance function and spectral density of the process $X_t$.

(10p)
3) A discrete time stochastic process, $X_t, t = 0, \pm 1, \pm 2, \ldots$, has the covariance function

$$r_X(\tau) = e^{-|\tau|/2}.$$

We have five (5) observation values of this process. The expected value of $X_t$ grows linearly with time and the growing speed, (the derivative), could be estimated simply by

$$\hat{\beta}_1 = \frac{[X_5 - X_1]}{4},$$

or by using linear regression

$$\hat{\beta}_2 = \frac{\sum_{i=1}^5 X_i \cdot (i - 3)}{\sum_{i=1}^5 (i - 3)^2}.$$

Compute the variances $V[\hat{\beta}_1]$ and $V[\hat{\beta}_2]$. Which of the two estimates of the growing speed, $\hat{\beta}_1$ or $\hat{\beta}_2$, is the most reliable to use, i.e., which has the smallest variance? (10p)

4) An AR(2)-process with

$$X_t + a_1 X_{t-1} + a_2 X_{t-2} = e_t,$$

has the parameters $a_1 = 1, a_2 = 0.5$. The process $e_t, t = 0, \pm 1, \pm 2, \ldots$, is white Gaussian noise with $E[e_t] = 0$ and $V[e_t] = 1$.

a) Calculate the covariance function $r_X(\tau)$, for $\tau = 0, \pm 1, \pm 2, \pm 3$. (10p)

b) What will happen with the parameter replacement, $a_2 = -0.5$? Find the roots of the characteristic equation of the corresponding AR(2)-process, i.e., when the parameters is replaced with $a_1 = 1, a_2 = -0.5$ and $V[e_t] = 1$. Also compute the variances $V[Y_2], V[Y_3]$ and $V[Y_4]$ from the corresponding iterative equation, $Y_t = -Y_{t-1} + 0.5Y_{t-2} + e_t$ when $Y_0 = Y_1 = 0$. Use the roots and the variances to explain the behavior of the process. (10p)

5) A continuous time Gaussian stationary stochastic process $X(t), t \in \mathbb{R}$, has the covariance function

$$r_X(\tau) = e^{-\alpha \tau^2/2},$$

where $\alpha = -200 \ln 0.9$.

a) Show that the process $X(t)$ is differentiable in quadratic mean. (5p)

b) Compute the probability,

$$P(X'(t) - \frac{X(t + 0.1) - X(t)}{0.1} > 1).$$

(15p)
6) Assume that a zero-mean stationary stochastic sequence $S_t$, $t = 0, \pm 1, \pm 2, \ldots$, is Gaussian and has the covariance function

$$r_S(\tau) = \begin{cases} 
4, & \tau = 0, \\
-1, & \tau = \pm 1, \\
0, & \text{for other values.}
\end{cases}$$

The signal $S_t$ is disturbed by zero-mean Gaussian colored noise $N_t$, $t = 0, \pm 1, \pm 2, \ldots$, with covariance function

$$r_N(\tau) = \begin{cases} 
4, & \tau = 0, \\
1, & \tau = \pm 1, \\
0, & \text{for other values.}
\end{cases}$$

The signal $S_t$ and noise $N_t$ are assumed to be independent.

a) Calculate the frequency function $H(f)$ and the impulse response $h_u$ for the optimal (Wiener) filter which, with $X_t = S_t + N_t$ as input received signal, and $Y_t = \sum_{u=-\infty}^{\infty} h_u X_{t-u}$ as output minimizes

$$E[(S_t - Y_t)^2].$$

Hint: The output should finally be expressed as $Y_t = aX_{t-1} + bX_t + cX_{t+1}$.

(6p)

b) If there is an "echo" in the received signal, the actual input sequence to the filter $h_u$ will instead be

$$X_t = S_t + 0.1S_{t-1} + N_t.$$ 

Find a system of equations, and solve for the coefficients $a, b, c$ so that

$$E[(S_t - (aX_{t-1} + bX_t + cX_{t+1})^2)],$$

is minimized for the input sequence with echo.

(14p)