1 The following figures show different realizations, covariance functions and spectral densities. The processes are either autoregressive, AR(\(p\)), or moving average, MA(\(q\)). Determine, with a motivation, what figures that are realizations, covariance functions and spectral densities. Also state which realization, covariance function and spectral density that are connected. Decide and motivate of which type the different processes are (AR or MA) and which orders, \(p\) or \(q\) they have. The orders can be assumed to be smaller than 4.

(10p)

2 Determine the covariance function and the spectral density of the MA(3)-process

\[X_t = e_t - e_{t-3},\]

where the Gaussian white noise process \(e_t\) has the variance \(V[e_t] = 2\).

(10p)
3 The figures below show the expected values of the spectrum estimates from two methods, the periodogram and the Welch method. The true process is an AR(6)-process and the number of data samples is \( n = 80 \). The periodogram uses a rectangle window and the Welch method applies 4 Hanning windows with 50 % overlap. Determine, with a short motivation, if the following statements are correct.

a) Using the Welch method gives a better resolution of close peaks.

b) The high sidelobes of the periodogram causes leakage and severe bias for frequencies \( f > 0.3 \).

c) The window lengths of the different Hanning windows of the Welch method described above is \( L = 20 \) (total data length \( n = 80 \)).

d) The standard deviation of the Welch spectrum estimate is reduced a factor 2 compared to the standard deviation of the periodogram.

e) Figure A shows the Welch method and figure B the periodogram.

![Figure A and Figure B](image)

4 A Gaussian stationary process \( X(t), t \in \mathbb{R} \), has expected value \( E[X(t)] = 2 \) and covariance function

\[
r_X(\tau) = e^{-\tau^2}.
\]

a) Compute the probability,

\[
P(X'(t) \geq X(t)).
\]

b) The Gaussian stationary process \( X(t) \), defined as above, is filtered through the linear, time-invariant filter with impulse response,

\[
h(t) = e^{-t^2/4}, \quad -\infty < t < \infty.
\]

Calculate the expected value, the covariance function and the spectral density for the output process.

![Graph](image)
We define the stationary Gaussian process $Y(t) = m + X(t)$, $t \in \mathbb{R}$, where $m$ is an unknown constant. The process $X(t)$ is assumed to have $E[X(t)] = 0$ and the spectral density

$$R_X(f) = \begin{cases} 1 & \text{for } 1 < |f| \leq 3, \\ 0 & \text{otherwise.} \end{cases}$$

An estimate of $m$ should be found as

$$\hat{m}_n = \frac{Y(d) + Y(2d) + \ldots + Y(nd)}{n},$$

by sampling the process $Y(t)$ with sample distance $d$.

a) Calculate the covariance function $r_Y(\tau)$. \hspace{1cm} (6p)

b) Determine the spectral density of the sampled process $Z_t = Y(t)$, $t = 0, \pm d, \pm 2d, \ldots$ for the sample distance $d = 1/2$. \hspace{1cm} (10p)

c) Determine the variance, $V[\hat{m}_n]$, when $d = 1/2$. \hspace{1cm} (4p)

The stationary Gaussian process $Y_t$, $t = 0, \pm 1, \pm 2, \ldots$, is the output of the causal linear time-discrete filter with the impulse response

$$h(t) = \begin{cases} 1 & t = 0, \\ 0.1(-0.7)^{t-1} & t \geq 1. \end{cases}$$

The input process $X_t$, $t = 0, \pm 1, \pm 2, \ldots$, is a sequence of independent $N(0, 1)$-variables.

a) Determine the cross-covariance $r_{X,Y}(\tau)$ and the cross-spectrum $R_{X,Y}(f)$ between the input and output processes. \hspace{1cm} (8p)

b) Determine the covariance function $r_Y(\tau)$ for the output process. \hspace{1cm} (12p)