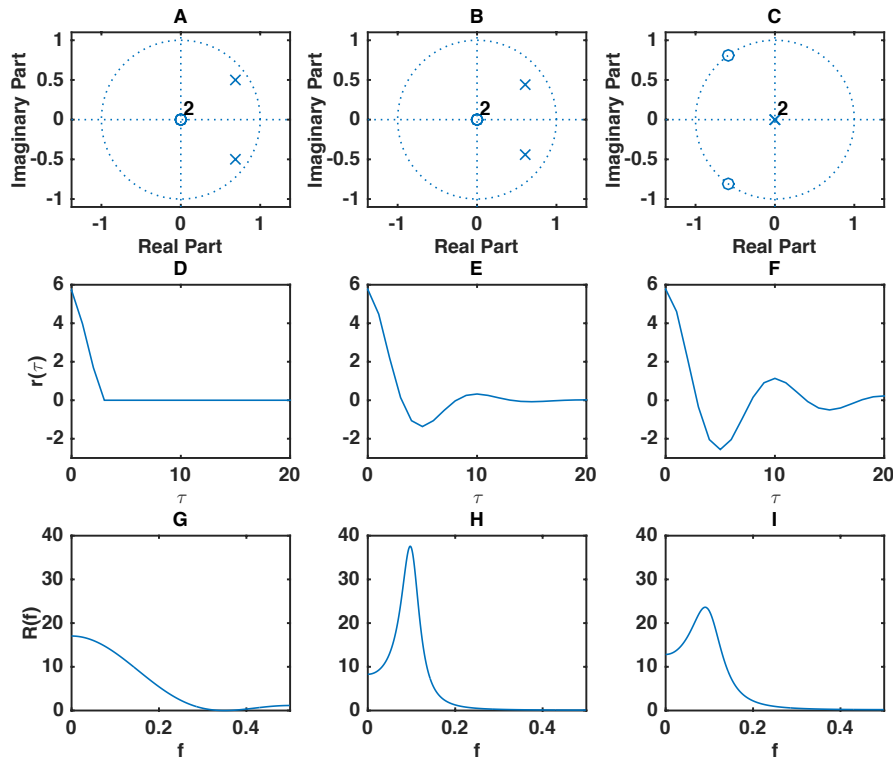


Exercise

The figures below belong to processes of type AR or MA. Determine which pole-zero-plots, covariance functions and spectral densities that correspond to the same process. Also decide the type of each process and the corresponding order.



Answer and explanation

The following figures correspond to the same processes: A-F-H which is an AR(2)-process, B-E-I which is an AR(2)-process, and C-D-G which is an MA(2)-process.

A pole-zero plot with all zeros at the origin and the poles located anywhere inside the unit circle corresponds to an AR-process. The number of poles corresponds to the order, in this case 2 in both A and B. A pole-zero plot with all poles at the origin and the zeros located elsewhere, as the one in C, corresponds to an MA-process and the number of zeros corresponds to the order, in this case 2.

An MA-process always has a covariance function where all values are zero for $\tau > \text{order}$, and thereby D has to correspond to C. An AR-process always has a covariance function of infinite length, where the period corresponds to the pole angle divided by 2π and the spectral peak frequency. The damping of the covariance function corresponds to the distance of the pole from the unit circle. Accordingly F belongs to A and E to B.

Spectral densities that include only peaks correspond to an AR-process. Stronger peaks correspond to a less damped covariance function and poles located closer to the unit circle, i.e., H-F-A and I-E-B. The spectral density in G corresponds to an MA-process as it includes only 'valleys' or, as in this case zeros, as the pole-zero plot zeros are located at the unit circle in C. Sometimes when the interpretation of the zeros is difficult, a spectral density plot in dB-scale is valuable, see exercise ARMA6.