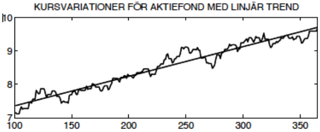
Another example of stock prices



The figure shows the daily stockprice of the 'Mathematical statistics special fund', april 2018-december 2018. The stock price at day t is modeled as

$$Y_t = b + a \cdot t + X_t, \qquad t = 100, 101, \ldots, 365,$$

where the linear trend increase $a = 0.00889$ and $b = 6.5$. The process

$$X_t = \frac{1}{16} \sum_{j=0}^{15} e_{t-j},$$

where the stationary Gaussian white noise process is defined with $e_t \in N(0, \sigma^2)$ and $\sigma^2 = 0.448$. Calculate the probability that the stock price increase from one day to another is larger than ten times the linear trend increase, i.e.,

$$P(Y_{t+1} - Y_t > 10 \cdot a).$$

Solution

The increase from day t to day $t+1$ is

$$Y_{t+1} - Y_t = b + a \cdot (t+1) + X_{t+1} - b - a \cdot t - X_t = a + X_{t+1} - X_t.$$

We simplify

$$\begin{aligned} X_{t+1} - X_t &= \frac{1}{16}\left(\sum_{j=0}^{15} e_{t+1-j} - \sum_{j=0}^{15} e_{t-j}\right) \\ &= \frac{1}{16}(e_{t+1} + e_t + \ldots + e_{t+1-15} - e_t - \ldots - e_{t-14} - e_{t-15}) \\ &= \frac{1}{16}(e_{t+1} - e_{t-15}), \end{aligned}$$

where most of the terms are cancelled out.

Solution

To compute the probability of a Gaussian process, we only need the expected value and the variance. We find the expected value

$$E[Y_{t+1} - Y_t] = a + E[X_{t+1} - X_t] = a,$$

as $E[X_{t+1} - X_t] = 0$. The variance is

$$V[Y_{t+1} - Y_t] = V[X_{t+1} - X_t] = \frac{1}{16^2} V[e_{t+1} - e_{t-15}] = \frac{2\sigma^2}{16^2}.$$

The probability is

$$\begin{aligned} P(Y_{t+1} - Y_t > 10a) &= P(a + X_{t+1} - X_t > 10a) = P(X_{t+1} - X_t > 9a), \\ &= 1 - P\left(X_{t+1} - X_t \leq 9a\right), \\ &= 1 - \Phi\left(\frac{16 \cdot 9a}{\sqrt{2}\sigma}\right) = 1 - \Phi(1.35242) = 0.088, \end{aligned}$$

with $a = 0.00889$ and $\sigma = \sqrt{0.448}$. The probability to sell at ten times the linear increase is 0.088, less than 10% chance. Change ten times to 5 times or 2 times and see the how much probability increase if you lower your claims.