Another example of stock prices

The figure shows the daily stock price of the 'Mathematical statistics special fund', April 2018-December 2018. The stock price at day $t$ is modeled as

$$Y_t = b + a \cdot t + X_t, \quad t = 100, 101, \ldots, 365,$$

where the linear trend increase $a = 0.00889$ and $b = 6.5$. The process

$$X_t = \frac{1}{16} \sum_{j=0}^{15} e_{t-j},$$

where the stationary Gaussian white noise process is defined with $e_t \in N(0, \sigma^2)$ and $\sigma^2 = 0.448$. Calculate the probability that the stock price increase from one day to another is larger than ten times the linear trend increase, i.e.,

$$P(Y_{t+1} - Y_t > 10 \cdot a).$$
Solution

The increase from day $t$ to day $t + 1$ is

$$Y_{t+1} - Y_t = b + a \cdot (t + 1) + X_{t+1} - b - a \cdot t - X_t = a + X_{t+1} - X_t.$$ 

We simplify

$$X_{t+1} - X_t = \frac{1}{16} \left( \sum_{j=0}^{15} e_{t+1-j} - \sum_{j=0}^{15} e_{t-j} \right)$$

$$= \frac{1}{16} \left( e_{t+1} + e_t + \ldots + e_{t+1-15} - e_t - \ldots - e_{t-14} - e_{t-15} \right)$$

$$= \frac{1}{16} \left( e_{t+1} - e_{t-15} \right),$$

where most of the terms are cancelled out.
To compute the probability of a Gaussian process, we only need the expected value and the variance. We find the expected value

\[ E[Y_{t+1} - Y_t] = a + E[X_{t+1} - X_t] = a, \]

as \( E[X_{t+1} - X_t] = 0 \). The variance is

\[ V[Y_{t+1} - Y_t] = V[X_{t+1} - X_t] = \frac{1}{16^2} V[e_{t+1} - e_{t-15}] = \frac{2\sigma^2}{16^2}. \]

The probability is

\[
P(Y_{t+1} - Y_t > 10a) = P(a + X_{t+1} - X_t > 10a) = P(X_{t+1} - X_t > 9a),
\]

\[ = 1 - P(X_{t+1} - X_t \leq 9a), \]

\[ = 1 - \Phi \left( \frac{16 \cdot 9a}{\sqrt{2}\sigma} \right) = 1 - \Phi(1.35242) = 0.088, \]

with \( a = 0.00889 \) and \( \sigma = \sqrt{0.448} \). The probability to sell at ten times the linear increase is 0.088, less than 10% chance. Change ten times to 5 times or 2 times and see the how much probability increase if you lower your claims.