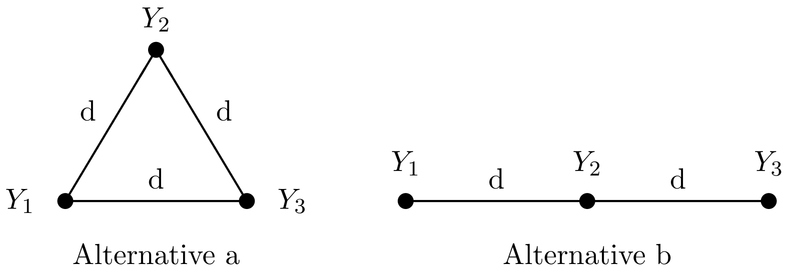

Figure 1: Two different microphone placements.

Exercise 2:14: Let Y_j, $j = 1, 2, 3$ be the measurements of the sound noise level of in a specific area. The dependence between measurements at different locations is described by $C[Y_i, Y_j] = r_Y(d) = e^{-d}$, where d is the distance between the i:th and j:th measurement point. The three microphones could be placed according to alternative a or b, see figure. Which placement gives the lowest variance of the average of the measurements?

Solution: The average of the three measurements is given as

$$\hat{m} = \frac{1}{3}(Y_1 + Y_2 + Y_3).$$

The variance for the average of alternative a is

$$\begin{aligned}
V[\hat{m}_a] &= C[\frac{1}{3}(Y_1 + Y_2 + Y_3), \frac{1}{3}(Y_1 + Y_2 + Y_3)] = \\
&= \frac{1}{3^2}\left(V[Y_1] + V[Y_2] + V[Y_3] + 2C[Y_1, Y_2] + 2C[Y_2, Y_3] + 2C[Y_1, Y_3]\right) = \\
&= \frac{1}{9}\left(1 + 1 + 1 + 2e^{-d} + 2e^{-d} + 2e^{-d}\right).
\end{aligned}$$

The variance for the average of alternative b is similarly

$$\begin{aligned}
V[\hat{m}_b] &= \frac{1}{3^2}\left(V[Y_1] + V[Y_2] + V[Y_3] + 2C[Y_1, Y_2] + 2C[Y_2, Y_3] + 2C[Y_1, Y_3]\right) = \\
&= \frac{1}{9}\left(1 + 1 + 1 + 2e^{-d} + 2e^{-d} + 2e^{-2d}\right).
\end{aligned}$$

We conclude that $V[\hat{m}_b] < V[\hat{m}_a]$, and therefore is b a better alternative to reduce the noise variance of the average measurement.